

# OPTIMIZATION

## The General Linear programming problem:

The general linear programming problem can be expressed as follows .

Find the value of variables  $X_1, X_2, \dots, X_n$  which (maximize or minimize) on objective function , which is a linear function of variable , such as

$$Z = C_1 X_1 + C_2 X_2 + \dots + C_n X_n$$

Subject to the constraints .

$$a_{11} X_1 + a_{12} X_2 + \dots + a_{1n} X_n (\leq, =, \geq) b_1$$

$$a_{21} X_1 + a_{22} X_2 + \dots + a_{2n} X_n (\leq, =, \geq) b_2$$

$$a_{31} X_1 + a_{32} X_2 + \dots + a_{3n} X_n (\leq, =, \geq) b_3$$

,

,

,

$$a_{m1} X_1 + a_{m2} X_2 + \dots + a_{mn} X_n (\leq, =, \geq) b_n$$

And meet the non-negative condition  $X_1, X_2, \dots, X_n \geq 0$  for each constraints one only one of signs ( $\leq, =, \geq$ ) holds but the sign may Vary from one constraints to on ther .

Because of the variety of notation is common use one finds the general L.P.P stated in mary forms some of there are :

## 1 ) Compact form by using the sign sign :

$$(\text{Maximize or Minimize}) Z = \sum_{j=1}^n C_j X_j$$

$$\sum_{j=1}^n a_{ij} X_j (\leq, =, \geq) b_i$$

And  $X_j \geq 0$  ,  $(j = 1, 2, \dots, n, i = 1, 2, \dots, m)$

## 2 ) Matrix \_ Vector forms :

$$(\text{Maximize or Minimize}) \quad CX$$

Subject to

$$Ax (\leq, =, \geq) b \quad \text{and } x \geq 0$$

Where A is a  $(m \times n)$  matrix , x is a  $(n \times 1)$  column vector , b is a  $(m \times 1)$  column vector and C is a  $(1 \times m)$  row vector in other words

$$A(m \times n) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$X(m \times 1) = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}, \quad b(m \times 1) = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \quad C(1 \times m) = [c_1 \quad c_2 \quad \cdots \quad c_n]$$

Where A is coefficient matrix , x is the decision variables b is the requirement vector , c is the cost (price or profit) vector of L.P.P .