Chapter Six

Hilbert Space

Definition : (Hilbert space)

The inner products space $(X, <.,.>)$ is said to be Hilbert space if X is a complete space under the norm induced by inner product.

Remark :

The Banach space is Hilbert space if the norm satisfied the parallelogram low.

Example 1: Show that R^n is Hilbert space .

Solution :

At first we prove that R^n is inner product space where $< x, y > = \sum_{i=1}^{n} x_i \, y_i$, $\forall x, y \in R^n$. (H.W.)

Now, we prove that R^n is complete space under the norm

$$\| x \| = \sqrt{< x, x >} = [\, \sum_{i=1}^{n} {x_i}^2 \,]^{\frac{1}{2}} \text{ (H.W.)}$$

Example 2: Let C[a, b] is Banach space under the norm define by $\|f\| = \sup_{x \in [a,b]} \{|f(x)|\}$, $\forall f \in C[a, b]$. Show that C[a, b] is not Hilbert space.

Solution :

C[a, b] is not Hilbert space since the norm is not satisfied the parallelogram low, as following : (H.W.)

<u>Example 3:</u> **Let the space ℓ^p , $p \geq 1$ is Banach space under the norm define by**

$\|x\|_p = [\sum_{i=1}^{\infty} |x_i|^p]^{\frac{1}{p}} \quad \forall\, x \in \ell^p$. **Show that ℓ^p is Hilbert space only if p=2.**

Solution:

<u>H.W</u>. Show that:

1- C^n is Hilbert space.

2- R^3 is Hilbert space.

3- Show that the Banach space $C[0,1]$ under the norm define by

$\|f\| = \sup_{x \in [0,1]} \{|f(x)|\}$, $\forall f \in C[0,1]$ **is not Hilbert space.**

4- Prove that every Hilbert space is Banach space ,but the convers is not true.

Chapter Seven

Orthogonal and Orthonormal