Chapter Two

Vector Space (Linear Space)

<u>Definition :</u> (vector space)

A vector space over a field F is a set V with two operations of addition $+: V \times V \to V$ and scalar multiplication $\cdot : F \times V \to V$ satisfying the following properties :-

1- $x + y \in V$, $\forall\, x, y \in V$

2- $x + y = y + x$, $\forall\, x , y \in V$

3- $(x + y) + z = x + (y + z)$, $\forall\, x, y , z \in V$.

4- $\exists\, 0 \in V$ such that $0 + x = x + 0$, $\forall\, x \in V$.

5- $\forall\, x \in V$, $\exists\, (-x) \in V$ such that $x + (-x) = (-x) + x = 0$.

6- $\alpha . x \in V$, $\forall\, x \in V$, $\alpha \in F$.

7- $\alpha\,(x + y) = \alpha\, x + \alpha y$, $\forall\, x, y \in V$, $\alpha \in F$

8- $(\alpha + \beta)x = \alpha\, x + \beta x$, $\forall\, \alpha, \beta \in F$, $x \in V$.

9- $(\alpha\, \beta)x = \alpha\, (\beta\, x)$.

10- $1 . x = x$ $\forall x \in V$

<u>Note :</u>

If F = R we say that V is a real vector space or (real linear space),

and if F= C we say that V is complex vector space or (complex linear space).

Example 1 :

1- R^n is linear space over a field R. H.W

2- Is Q linear space over a field R ? H.w

Definition : (subspace)

A subset W of a vector space V over a field F is a **subspace** of V if W itself is a vector space over a field F.

Note: the every non-zero vector space V has at least two subspace $\{0\}$ and V itself.

Theorem :

Let W be a **non empty subset** of a vector space V over a field F , then W is a subspace of V if and only if:

1- $x + y \in W \ , \forall x, y \in W$

2- $\alpha x \in W \ , \forall x \in W \ , \alpha \in F$

Example 2 : let $W = \{(a, b, 0): a, b \in R\} \subseteq R^3$.

Show that W is a subspace of R^3.

Solution : $\because (0,0,0) \in W, \ \therefore W \neq \emptyset$

1- Let $v_1, v_2 \in W$

$\therefore v_1 = (a_1, b_1, 0) , \qquad a_1, b_1 \in R$

$v_2 = (a_2, b_2, 0) , \qquad a_2, b_2 \in R$

$v_1 + v_2 = (a_1, b_1, 0) + (a_2, b_2, 0)$

$= (a_1 + a_2, b_1 + b_2, 0 + 0)$

$\because R$ is vector space

$\therefore a_1 + a_2 \in R \ \& \ b_1 + b_2 \in R$

$\therefore v_1 + v_2 \in W$

2- Let $v \in W$

$v = (a, b, 0)$, $a, b \in R$ and let α scalar.

$\alpha v = \alpha(a, b, 0) = (\alpha a, \alpha b, 0)$

$\because R$ is vector space

$\therefore \alpha a, \alpha b \in R$

$\therefore \alpha v \in W$

<u>Example 3:</u>

Let $w = \{(a, b, 1): a, b \in R\} \subseteq R^3$. Is W subspace of R^3.

Solution $\because (0,0,1) \in W$, $\therefore W \neq \emptyset$.

Let $v_1, v_2 \in W$

$\therefore v_1 = (a_1, b_1, 1)$, $a_1, b_1 \in R$

$v_2 = (a_2, b_2, 1)$, $a_2, b_2 \in R$

$v_1 + v_2 = (a_1, b_1, 1) + (a_2, b_2, 1)$

$= (a_1 + a_2 , b_1 + b_2 , 1 + 1)$

$= (a_1 + a_2 , b_1 + b_2 , 2)$

$\therefore v_1 + v_2 \notin W$

$\therefore W$ is not subspace

Some principle concepts in vector space

Some principle concepts in vector space

<u>Definition :</u> (Convex set)