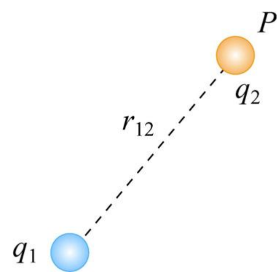
Potential Energy in a System of Charges

1. If a system of charges (two charges q_1 and q_2) is assembled by an external agent. The potential due to q_1 at a point P be V_1 (as shown in figure)



The work done by an agent in bringing the second charge q_2 from infinity to

P is $W_2 = q_2 V_1$ ($W_1 = 0$ no work is required to set up the first charge)

The potential energy of the system is

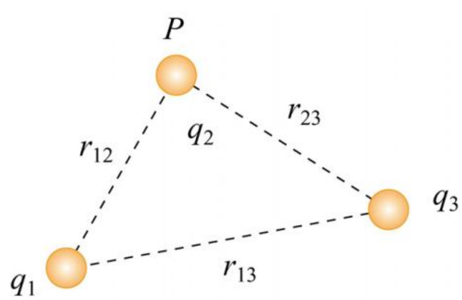
$$U_{12} = W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

Where $V_1 = q_1 / 4\pi\varepsilon_0 r_{12}$ and r_{12} is the distance measured from q_1 to *P*.

If and q_1 and q_2 have the same sign ($U_{12} > 0$), positive work must be done to overcome the electrostatic repulsion.

if the signs are opposite ($U_{12} < 0$), due to the attractive force between the charges.

2. If a system has three charges (To add a third charge q_3 to the previous system) as shown in figure,



the work required is

$$W_3 = q_3(V_1 + V_2) = \frac{q_3}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

The potential energy of this configuration is then

$$U = W_2 + W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) = U_{12} + U_{13} + U_{23}$$

The equation shows that the total potential energy is simply the sum of the contributions from distinct pairs.

Generalizing to a system of *N* charges, we have

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j>i}}^{N}\frac{q_i q_j}{r_{ij}}$$

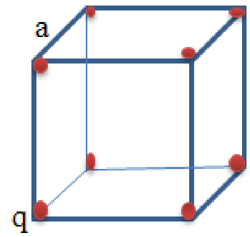
Note: where the constraint *ji*> is placed to avoid double counting each pair. Alternatively, one may count each pair twice and divide the result by 2. This leads to

$$U = \frac{1}{8\pi\varepsilon_0}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N}\frac{q_i q_j}{r_{ij}} = \frac{1}{2}\sum_{i=1}^{N} q_i\left(\frac{1}{4\pi\varepsilon_0}\sum_{\substack{j=1 \\ j\neq i}}^{N}\frac{q_j}{r_{ij}}\right) = \frac{1}{2}\sum_{i=1}^{N} q_i V(r_i)$$

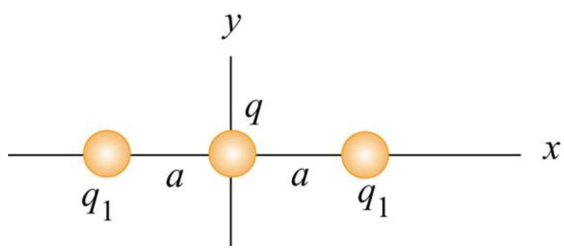
Where $V(r_i)$, the quantity in the parenthesis, is the potential at $\vec{r}_i$ (location of q_i) due to all the other charges.

<u>H. W.</u>

1. How much work is done to assemble eight identical point charges, each of magnitude *q*, at the corners of a cube of side *a*?



2. Three charges with q= 3×10^{-18} C and q_1= 6×10^{-6} C are placed on the *x*-axis, as shown in the figure. The distance between q and q_1 is a = 0.600 m.



(a) What is the net force exerted on q by the other two charges q_1?

(b) What is the electric field at the origin due to the two charges q_1?

(c) What is the electric potential at the origin due to the two charges q_1?

3.

Two charges $q_1 = 3.0\,\mu C$ and $q_2 = -4.0\,\mu C$ initially are separated by a distance $r_0 = 2.0\,cm$. An external agent moves the charges until they are $r_f = 5.0\,cm$ apart.

(a) How much work is done by the *electric field* in moving the charges from r_0 to r_f? Is the work positive or negative?

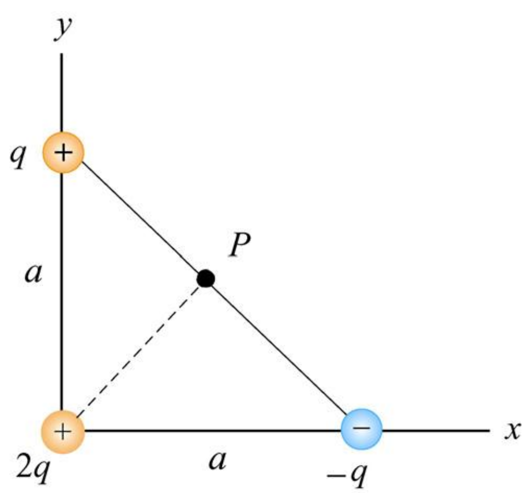
(b) How much work is done by the *external agent* in moving the charges from r_0 to r_f? Is the work positive or negative?

(c) What is the potential energy of the initial state where the charges are $r_0 = 2.0\,\text{cm}$ apart?

(d) What is the potential energy of the final state where the charges are $r_f = 5.0\,\text{cm}$ apart?

(e) What is the change in potential energy from the initial state to the final state?

4. A right isosceles triangle of side *a* has charges *q*, +2*q* and −*q* arranged on its vertices, as shown in Figure

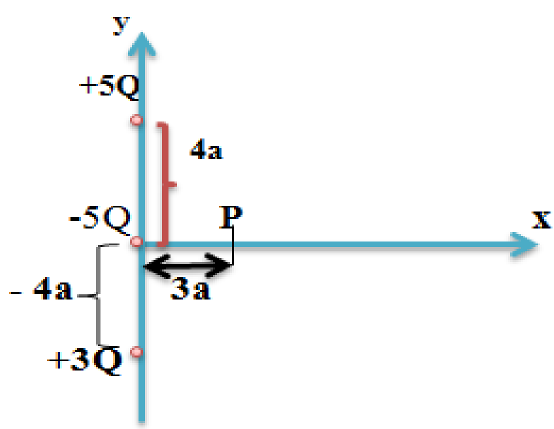


(a) What is the electric potential at point P, midway between the line connecting the $+q$ and $-q$ charges, assuming that $V = 0$ at infinity? [Ans: $q/\sqrt{2}\ \pi\varepsilon_0 a$.]

(b) What is the potential energy U of this configuration of three charges? What is the significance of the sign of your answer? [Ans: $-q^2/4\sqrt{2}\ \pi\varepsilon_0 a$, the negative sign means that work was done on the agent who assembled these charges in moving them in from infinity.]

(c) A fourth charge with charge $+3q$ is slowly moved in from infinity to point P. How much work must be done in this process? What is the significance of the sign of your answer? [Ans: $+3q^2/\sqrt{2}\ \pi\varepsilon_0 a$, the positive sign means that work was done by the agent who moved this charge in from infinity.]

5. Three charges, $+5Q$, $-5Q$, and $+3Q$ are located on the y-axis at $y = +4a$, $y = 0$, and $y = -4a$, respectively. The point P is on the x-axis at $x = 3a$.



(a) How much energy did it take to assemble these charges?

(b) What are the x, y, and z components of the electric field $\vec{E}$ at P?

(c) What is the electric potential V at point P, taking $V = 0$ at infinity?

(d) A fourth charge of $+Q$ is brought to P from infinity. What are the x, y, and z components of the force $\vec{F}$ that is exerted on it by the other three charges?

(e) How much work was done (by the external agent) in moving the fourth charge $+Q$ from infinity to P?