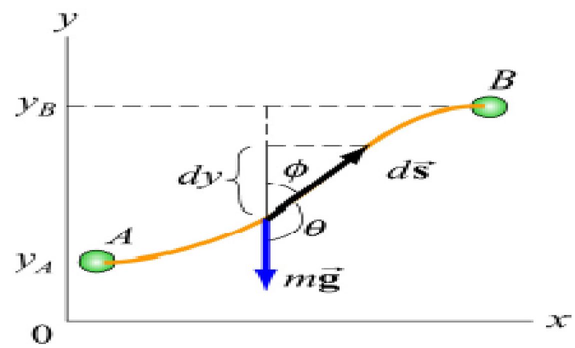
Electric Potential

Potential and Potential Energy

The work done by gravity in moving an object from height y_A *to* y_B as shown in figure is

$$W_g = \int \vec{\mathbf{F}}_g \cdot d\vec{\mathbf{s}} = \int_A^B mg\cos\theta\, ds = -\int_A^B mg\cos\phi\, ds = -\int_{y_A}^{y_B} mg\, dy = -mg(y_B - y_A)$$



The result is independent of the path, and is only a function of the change in vertical height $y_B - y_A$.
Note: When the object moves around and then returns to where it starts off, the net work done by the gravitational field would be zero, and we say that the gravitational force is conservative. The line integral of force around a closed loop vanishes:

$$\oint \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = 0$$

The change in potential energy associated with a conservative force $\vec{F}$ acting on an object as it moves from *A* to *B* is defined as:

$$\Delta U = U_B - U_A = -\int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = -W$$

where *W* is the work done by the force on the object.

In the presence of an electric field $\vec{E}$, we define the electric potential difference between two points A and B as

$$\Delta V = -\int_A^B (\vec{\mathbf{F}}_e / q_0) \cdot d\vec{\mathbf{s}} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

where q_0 is a test charge. The potential difference ΔV represents the amount of work done per unit charge to move a test charge q_0 from point A to B, without changing its kinetic energy.
The two quantities (electric potential and electric potential energy) are related by

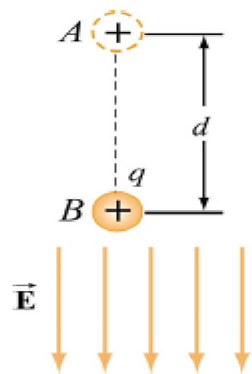
$$\Delta U = q_0 \Delta V$$

The SI unit of electric potential is volt (V):
1volt =1joule/coulomb (1 V= 1 J/C)
at the atomic or molecular scale → electron volt (eV)
Electron Volt(eV): the energy an electron acquires (or loses) when moving through a potential difference of one volt:

$$1\,\text{eV} = (1.6\times10^{-19}\,\text{C})(1\,\text{V}) = 1.6\times10^{-19}\,\text{J}$$

Electric Potential in a Uniform Field

When a charge $+q$ moving in the direction of a uniform electric field $\vec{E} = E_0(-\hat{j})$, as shown in Figure.



the potential difference between points A and B is given by

$$\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}}\cdot d\,\vec{\mathbf{s}} = -E_0\int_A^B ds = -E_0 d < 0$$

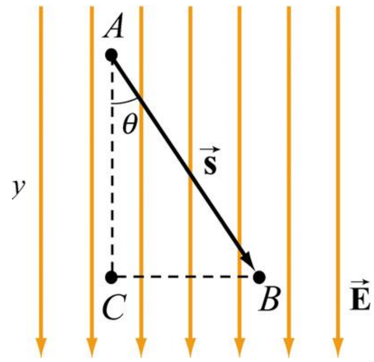
Point B is at a lower potential compared to A.
The electric field lines always point from higher potential to lower.

The change in potential energy is

$$\Delta U = U_B - U_A = -qE_0 d$$

Since $q > 0 \rightarrow \Delta U < 0$ (the potential energy of a positive charge decreases as it moves along the direction of the electric field.)

if the path from A to B is not parallel to$\vec{E}$, but instead at an angle θ, as shown in Figure.



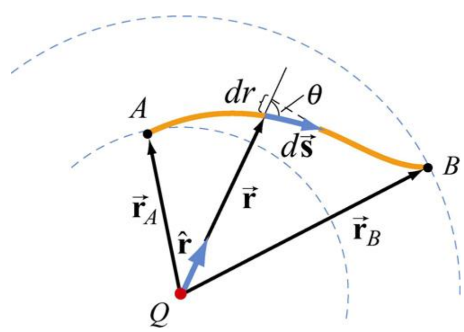
The potential difference becomes

$$\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\vec{\mathbf{E}} \cdot \vec{\mathbf{s}} = -E_0 s \cos\theta = -E_0 y$$

The moving along the direction of the electric field $\vec{E}$ leads to a lower electric potential.

Electric Potential due to Point Charges

Determining the potential difference between two points A and B due to a charge $+Q$.



$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\,.d\vec{s}$$

$$\Delta V = V_B - V_A = -\int_A^B \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \cdot d\,\vec{\mathbf{s}}$$

The electric field produced by Q is

$\vec{\mathbf{E}} = (Q/4\pi\varepsilon_0 r^2)\hat{\mathbf{r}}$ where $\hat{r}$ is a unit vector pointing toward the field point.

$$\Delta V = V_B - V_A = -\int_A^B \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}\cdot d\vec{s} = -\int_A^B \frac{Q}{4\pi\varepsilon_0 r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Where $\hat{\mathbf{r}}\cdot d\vec{s} = ds\cos\theta = dr$

The potential difference ΔV depends only on the endpoints, independent of the choice of path taken.

The electric potential at a point P becomes

$$V_P = -\int_\infty^P \vec{\mathbf{E}}\cdot d\vec{s}$$

To choose the reference point to be at infinity (potential equals zero).

With this reference, the electric potential at a distance r away from a point charge Q becomes

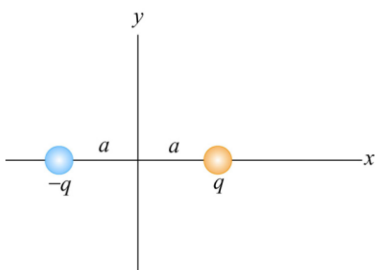
$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$ (one point charge)

For more than one point charge, the total electric potential is simply the sum of potentials due to individual charges:

$$V(r) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i} = k_e\sum_i \frac{q_i}{r_i}$$ (more than one point charge)

Example

Consider a system of two charges shown in Figure.



Find the electric potential at an arbitrary point on the *x* axis.

Solution:

The electric potential at a point on the *x* axis, we have

$$V(x) = \frac{1}{4\pi\varepsilon_0}\frac{q}{|x-a|} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{|x+a|} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{|x-a|} - \frac{1}{|x+a|}\right]$$