Dipole in Electric Field

❖ When we place an electric dipole in a uniform field $\vec{E} = E\hat{\imath}$.

- The dipole moment vector $\vec{P}$ making an angle with the *x*-axis.
- the unit vector which points in the direction of $\vec{P}$ is
- $\hat{p} = \cos\theta\hat{\imath} + \sin\theta\hat{\jmath}$

  $\vec{P} = 2aq\,\hat{p}$ (Where **2a= d**)

  $$\vec{p} = 2qa(\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}})$$

- The net force on the dipole is

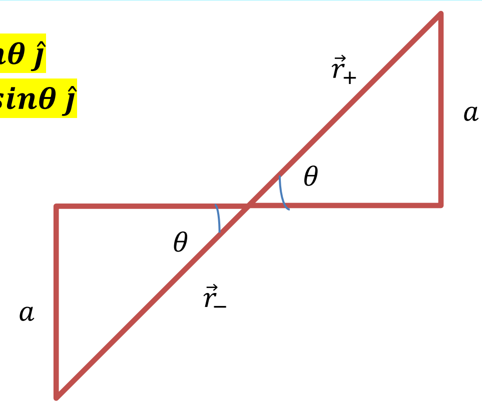
$$\vec{F}_{net} = \vec{F}_+ + \vec{F}_- = 0$$

  - $\vec{F}_+ \rightarrow$ Positive charge experiences force due to the field.
  - $\vec{F}_- \rightarrow$ Negative charge experiences force due to the field.

- The torque about the midpoint *O* of the dipole is

$$\vec{\tau} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_- = (a\cos\theta\,\hat{\mathbf{i}} + a\sin\theta\,\hat{\mathbf{j}}) \times (F_+\,\hat{\mathbf{i}}) + (-a\cos\theta\,\hat{\mathbf{i}} - a\sin\theta\,\hat{\mathbf{j}}) \times (-F_-\,\hat{\mathbf{i}})$$

  - $\vec{r}_+ = a\,\hat{r}_+$ where $\hat{r}_+ = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$
  - $\vec{r}_- = a\,\hat{r}_-$ where $\hat{r}_- = -\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}$
  - $\vec{F}_+ = F_+\hat{\imath}$ and $\vec{F}_+ = -F_-\,i$

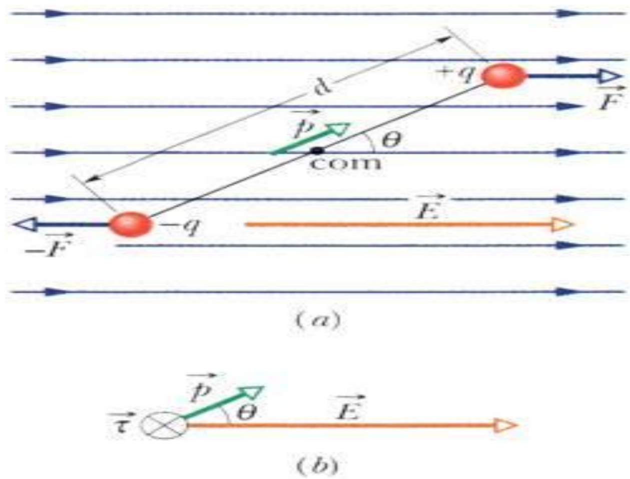


$\vec{\tau} = a\sin\theta\,F_+(-\hat{k}) + a\sin\theta\,F_-(-\hat{k})$

= $2a\,F\sin\theta\,(-\hat{k})$ (The direction of the torque is into the page)

  - Where we have used $F_+ = F_- = F$
    - The magnitude of the torque is

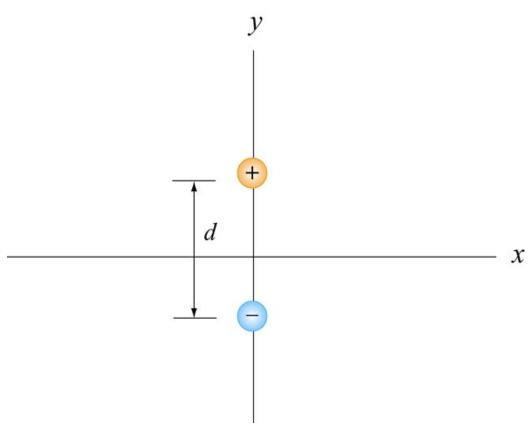
- $\tau = 2a\,(qE)\sin\theta = (2aq)\,E\sin\theta = PE\sin\theta$ (Where F=qE and 2aq= P)
- The general expression for torque becomes
  - $\vec{\tau} = \vec{P} \times \vec{E}$ (The cross product of the dipole moment with the electric field is equal to the torque).



(a) An electric dipole in a uniform external electric field E. Two centers of equal but opposite charge are separated by distance d. The line between them represents the rigid connection. (b) Field E causes a torque tau on the dipole. The direction of the torque is into the page, as represented by the arrow tail symbol.

H.W

An electric dipole consists of two charges $q_1 = +2e$ and $q_2 = -2e$ ($e = 1.6\times10^{-19}$ C), separated by a distance $d = 10^{-9}$ m. The electric charges are placed along the y-axis as shown in figure.



Suppose a constant external electric field $\vec{E}_{ext} = (3\hat{\imath} + 3\hat{\jmath})N/C$ is applied.

(a) What is the magnitude and direction of the dipole moment?

(b) What is the magnitude and direction of the torque on the dipole?

Potential Energy of an Electric Dipole

- The work done by the electric field (is stored as its potential energy) to rotate the dipole by an angle $d\theta$ is

$$dW = -\tau d\theta = -pE\sin\theta\, d\theta$$

  - The negative sign indicates that the torque *opposes* any increase in θ.

- The total amount of work done by the electric field to rotate the dipole from an angle $\theta_0 to\ \theta\ is$

$$W = \int_{\theta_0}^{\theta} (-pE\sin\theta)d\theta = pE\left(\cos\theta - \cos\theta_0\right)$$

  - If $cos\theta\ > cos\theta_0$ → a *positive* work is done by the field.

- The change in potential energy ΔU of the dipole is the negative of the work done by the field:

$$\Delta U = U - U_0 = -W = -pE\left(\cos\theta - \cos\theta_0\right)$$

  - Where $U_0 = \ -PEcos\theta_0$ is the potential energy at a reference point ($\theta_0 = \pi/2$).
    - The potential energy is zero at a reference point→ $U_0 = 0\ (\theta_0 = \pi/2)$.

- in the presence of an external field the electric dipole has a potential energy

$$U = -pE\cos\theta = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}}$$

    - when the dipole $\vec{p}$ is aligned parallel to $\vec{E}$ → U a minimum with($U_{min} = -PE$)
      → the system is at a stable equilibrium.
    - when $\vec{p}$ and $\vec{E}$ are anti-parallel → **U** a maximum with ($U_{min} = +PE$)
      → the system is unstable.

The P.E. of an electric dipole in an E-field