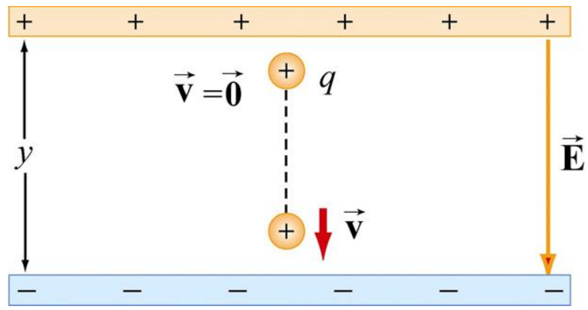
Force on a Charged Particle in an Electric Field

- A charge +q moving between two parallel plates of opposite charges, as shown in Figure.



- The electric field (constant) between the plates is $\vec{E} = -E_y\,\hat{j}$.
- The charge will experience a downward Coulomb force
  $\vec{F}_e = q\,\vec{E}$
- According to Newton's second law, a net force will cause the charge to accelerate with an acceleration. Suppose the particle is at rest (v_{0y}=0).
  $\vec{a} = \frac{\vec{F}_e}{m} = \frac{q\,\vec{E}}{m} = -\frac{qE_y}{m}\hat{j} \rightarrow |a| = \frac{qE_y}{m} = |a_y|$
- The final speed v of the particle as it strikes the negative plate is

$$v_y^2 = v_{0y}^2 + 2a_y y$$

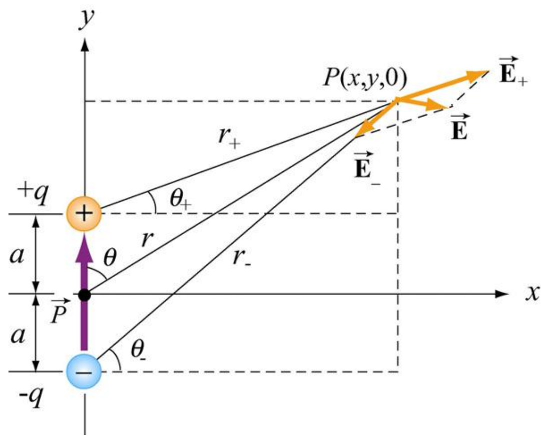
$$v_y^2 = 2a_y y \text{ (Where } v_{0y} = 0) \rightarrow v_y = \sqrt{2|a_y|y} = \sqrt{2\frac{qE_y}{m}y}$$

- The kinetic energy of the particle when it strikes the plate is

$$K = \frac{1}{2}m\,v_y^2 = qE_y\,y$$

Electric Dipole

An electric dipole consists of two equal but opposite charges, + q and $-q$, separated by a distance 2a , as shown in Figure.



- The dipole moment vector $\vec{p}$ which points from−q to +q (in the +y- direction) is given by

$$\vec{\mathbf{p}} = 2qa\,\hat{\mathbf{j}}$$

- The magnitude of the electric dipole is

$$p = 2qa\,,\ \text{where}\ \ q > 0$$

- For an overall charge-neutral system having N charges, the electric dipole vector $\vec{p}$ is defined as

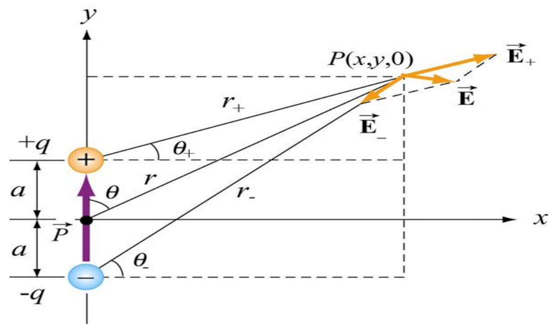
$$\vec{\mathbf{p}} \equiv \sum_{i=1}^{i=N} q_i \vec{\mathbf{r}}_{\mathbf{i}}$$

  - Where $\vec{r}_i$ is the position vector of the charge q_i.

- ***Examples of dipoles include HCL, CO, H_2O and other polar molecules.***

The Electric Field of a Dipole

What is the electric field strength at the point P?



The electric field due to positive charge (+q) is

$$\vec{E}_+ = E_{+(x)}\hat{\imath} + E_{+(y)}\hat{\jmath}$$

$$\vec{E}_+ = E_+ \cos\theta_+ \,\hat{\imath} + E_+ \sin\theta_+ \,\hat{\jmath}$$

$$\vec{E}_+ = k\,\frac{q}{{r_+}^2}\,\frac{x}{r_+}\,\hat{\imath} + k\,\frac{q}{{r_+}^2}\,\frac{y-a}{r_+}\hat{\jmath}$$ Where $\cos\theta_+ = \frac{x}{r_+}$, $\sin\theta_+ = \frac{y-a}{r_+}$, $E_+ = \frac{q}{{r_+}^2}$

$$\vec{E}_+ = k\,\frac{qx}{{r_+}^3}\,\hat{\imath} + k\,\frac{q(y-a)}{{r_+}^3}\,\hat{\jmath}$$

$$\boxed{\vec{E}_+ = k\,\frac{q}{{r_+}^3}\,[x\,\hat{\imath} + (y-a)\,\hat{\jmath}]}$$

The electric field due to negative charge (- q) is

$$\vec{E}_- = E_{-(x)}(-\hat{\imath}) + E_{-(y)}(-\hat{\jmath})$$

$$\vec{E}_- = E_- \cos\theta_-\,(-\hat{\imath}) + E_- \sin\theta_-\,(-\hat{\jmath})$$

$$\vec{E}_- = E_-[\cos\theta_-\,(-\hat{\imath}) + \sin\theta_-\,(-\hat{\jmath})]$$

$$\boxed{\vec{E}_- = \frac{kq}{{r_-}^3}[x(-\hat{\imath}) + (y+a)(-\hat{\jmath})]}$$ Where $\cos\theta_- = \frac{x}{r_-}$, $\sin\theta_- = \frac{(y+a)}{r_-}$, $E_- = \frac{kq}{{r_-}^2}$

The x-component of the electric field strength at the point P is

$$E_x = E_{+(x)} + E_{-(x)}$$

$$\boxed{E_x = kqx\left\{\frac{1}{[x^2+(y-a)^2]^{3/2}} - \frac{1}{[x^2+(y+a)^2]^{3/2}}\right\}}$$ Where $r_- = \sqrt{x^2 + (y+a)^2}$

$r_+ = \sqrt{x^2 + (y-a)^2}$

From the figure we have $x = r\sin\theta\ , y = r\cos\theta\ , x^2 + y^2 = r^2$

$$[x^2 + (y \pm a)^2]^{-3/2} = [x^2 + y^2 + a^2 \pm 2ay]^{-3/2}$$

$$= [r^2 + a^2 \pm 2ay]^{-3/2}$$

$$= [r^2(1 + \frac{a^2 \pm 2ay}{r^2})]^{-3/2}$$

$$= r^{-3}[(1 + \frac{a^2 \pm 2ay}{r^2})]^{-3/2}$$

When $r \gg a \rightarrow$ we use Taylor series expansion ($s \equiv \frac{a^2 \pm 2ay}{r^2}$)

$$(1+s)^{-3/2} = 1 - \frac{1}{1!}\frac{3}{2}s + \frac{1}{2!}\left(\frac{-3}{2}\cdot\frac{-5}{2}\right)s^2 - \cdots\cdots\cdots\cdots$$

$= 1 - \frac{3}{2}s$ Where the second term is neglect

$$r^{-3}[(1 + \frac{a^2 \pm 2ay}{r^2})]^{-3/2} = r^{-3}\ \left[1 - \frac{3}{2}(\frac{a^2 \pm 2ay}{r^2})\right]$$

$$E_x = kqr\sin\theta\left\{r^{-3}\left[1 - \frac{3}{2}\left(\frac{a^2 - 2ay}{r^2}\right)\right] - \left[1 - \frac{3}{2}\left(\frac{a^2 + 2ay}{r^2}\right)\right]\right\}$$

$$E_x = kqr^{-2}\sin\theta\ \frac{6ay}{r^2}$$

$$\boxed{E_x = \frac{3kP}{r^3}\sin\theta\cos\theta}$$ Where $y = r\cos\theta\ , P = 2a\cdot q$

The y-component of the electric field strength at the point P is

$$E_y = E_{+(y)} + E_{-(y)}$$

$$E_y = kq\left\{\frac{y-a}{[x^2+(y-a)^2]^{3/2}} - \frac{y+a}{[x^2+(y+a)^2]^{3/2}}\right\}$$

$$\boxed{E_y = \frac{kP}{r^3}\ (3\cos^2\theta - 1)}$$

The net electric field strength at the point P is

$$E = \sqrt{{E_x}^2 + {E_y}^2}$$

$$\boxed{E = \sqrt{\left(\frac{3kP}{r^3}\ \sin\theta\cos\theta\right)^2 + \left(\frac{kP}{r^3}\ (3\cos^2\theta - 1)\right)^2}}$$