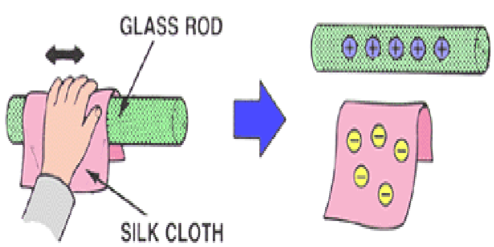
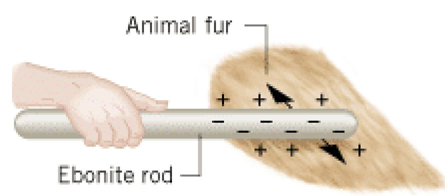
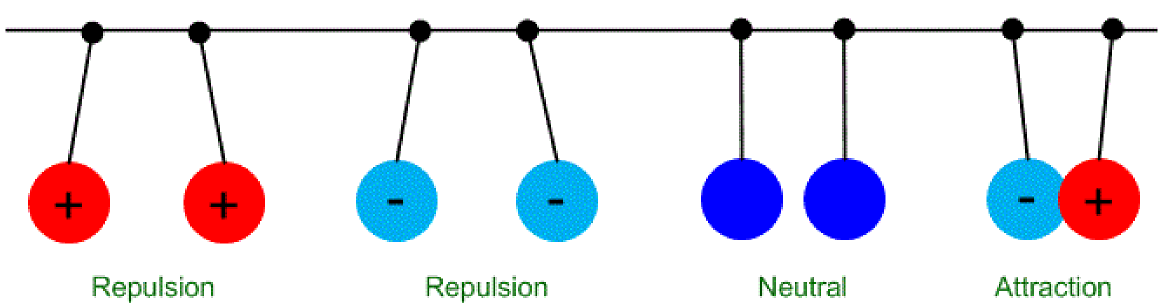
Coulomb's Law

Electric Charge

- There are two types of observed electric charges (positive and negative).
- A glass rod rubbed with silk gets positively charged.
- An ebonite rod rubbed with fur gets negatively charged.
- Like charges repel and opposite charges attract each other.
- The total amount of charge is conserved since charge can neither be created nor destroyed.
- A charge can be transferred from one body to another.
- The charge of an electron or proton (e= 1.602×10^{-19} C).







The Laws of Attraction and Repulsion

Coulomb's Law:

- It deals with two point charges, separated by a distance r in vacuum.
- The force exerted by q_1 on q_2 is given by Coulomb's law:

$$\vec{F}_{12} = K_e \frac{q_1 q_2}{r^2}\hat{r} \quad \text{(Coulomb's law)}, \quad K_e = \frac{1}{4\pi\varepsilon_0}$$

  - The electric (electrostatic) force is a vector which has both magnitude and direction.
  - The electric force acts at a distance, even when the objects are not in contact with one another.
  - $F_{12} = F_{21}$, $\vec{F}_{12} = -\vec{F}_{21}$ (Newton's third law)

$\hat{r}$: Unit vector directed from q_1 to q_2.

$K_e = \frac{1}{4\pi\varepsilon_0}$ ⟶ Coulomb's constant = 8.99 x 10^9 $N.m^2/C^2$

ε_0 : Permittivity of free space = $8.85\ x\ 10^{-12}\ \frac{C^2}{N.m^2}$

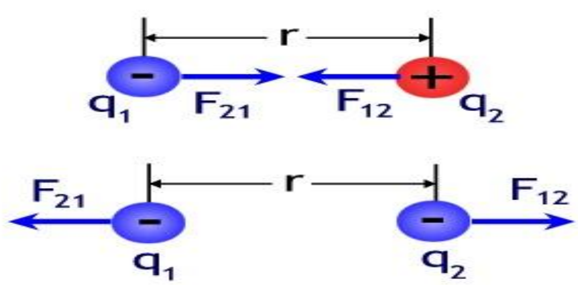
F_{12} → Magnitude of the force of attraction or repulsion [N]

q_1 → Magnitude of one particle's charge in Coulomb's [C]

q_2 → Magnitude of the other particle's charge in Coulomb's [C]

r → Distance between the particles [m]

- The electrostatic force(F_e) >> the gravitational force (F_g),
  i.e. the gravitational effect can be neglected when dealing with electrostatic forces. For example a Hydrogen atom (a distance between the electron and the proton r = 5.3 x 10^{-11} m), F_e= 8.2 x 10^{-8} N , F_g= 3.6 x 10^{-47} N.

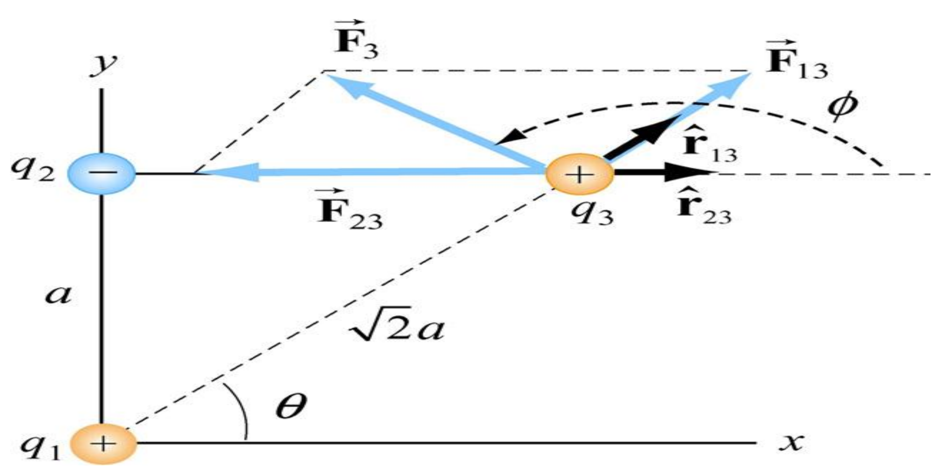


(Coulomb interaction between two charges)

Principle of Superposition(more than two charges)

- The net force on any one charge is simply the vector sum of the forces exerted on it by the other charges. For a system of N charges, the net force ($\vec{F}_j$) experienced by the jth particle would be:
- $\vec{F}_j = \sum_{\substack{i=1 \\ i\neq j}}^{N} \vec{F}_{ij}$
  - $\vec{F}_{ij}$ Denotes the force between particles i and j.
- If the charges are in fixed positions, then the net force between any two charges is independent of the presence of other charges.

Example: Three charges are arranged as shown in Figure. Find the force on the charge q_3, assuming that $q_1= +6.0 \times 10^{-6}$C, $q_2= -q_1$, $q_3= + 3.0 \times 10^{-6}$C and $a= 2 \times 10^{-2}$m.



Solution: Using the superposition principle, the force on q_3 is

$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + \frac{q_2 q_3}{r_{23}^2}\hat{r}_{23}\right)$ (since q_2 is negative)

$\hat{r}_{13} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath} = \frac{1}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})$ (Where $\sin\theta = \cos\theta$)

$\hat{r}_{23} = -\hat{\imath}$, $r_{13} = \sqrt{2}\,a$, $r_{23} = a$, $q_2 = - q_1$

The total force is

$$\vec{F}_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} - \frac{q_2 q_3}{r_{23}^2}\hat{r}_{23}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{(\sqrt{2}\,a)^2}\frac{1}{\sqrt{2}}(\hat{\imath} + \hat{\jmath}) - \frac{q_1 q_3}{a^2}\hat{\imath}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{(a)^2}[(\frac{\sqrt{2}}{4} - 1)\hat{\imath} + \frac{\sqrt{2}}{4}\hat{\jmath})]$$

The magnitude of the total force is given by

$$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{(a)^2}[(\sqrt{2}/4 - 1)^2 + (\frac{\sqrt{2}}{4})^2]^{\frac{1}{2}}$$

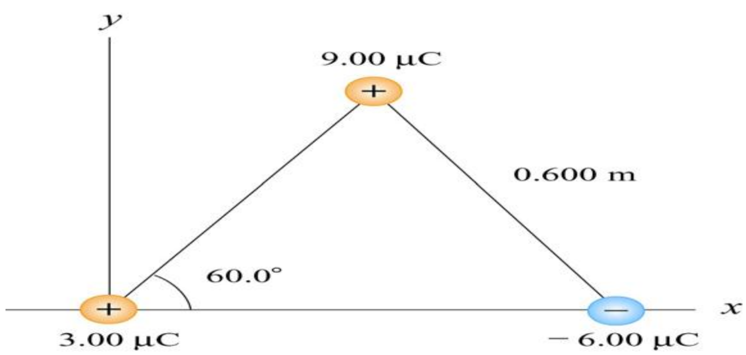
$$= (9.0\ X\ 10^9 N.m^2/C^2)\frac{(6\ X10^{-6}C)(3\ X10^{-6}C)}{(2\ X\ 10^{-2}m)^2}(0.74) = 3.0\ N$$

The angle that the force makes with the positive -axis is

$$\emptyset = \tan^{-1}(\frac{F_{3y}}{F_{3x}}) = tan^{-1}(\frac{\sqrt{2}/4}{-1+\sqrt{2}/4}) = 151.3°$$

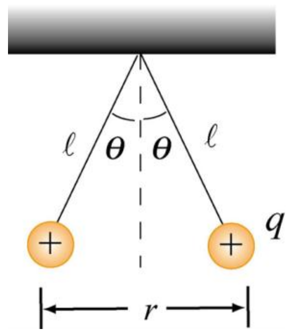
H.W:

1. Three point charges are placed at the corners of an equilateral triangle, as shown in Figure.



Calculate the net electric force experienced by (a) the 9.00 µC charge, and (b) The - 6.00 µC charge.

2. Two tiny conducting balls of identical mass *m* and identical charge *q* hang from non-conducting threads of length *l*. Each ball forms an angle θ with the vertical axis, as shown in Figure.



Show that, at equilibrium, the separation between the balls is

$$r = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{1/3}$$