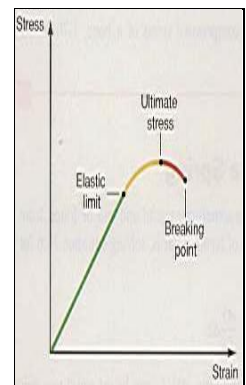
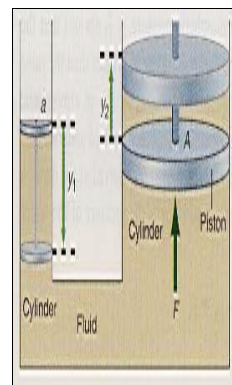
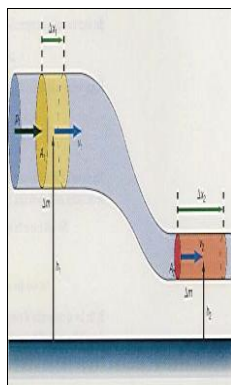
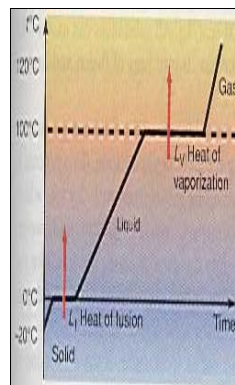
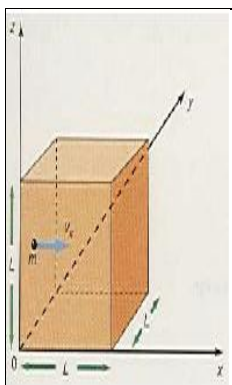


Kirkuk University

Science College

Physics Department

***Lectures of***  
***Properties of Matter***  
***Lecture 6***  
***The Ideal gas law***



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Lecture 6 ( The ideal gas law )	
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### 6.1 The Ideal Gas Law

The three gas laws,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad p = \text{constant} \quad (17.21)$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad V = \text{constant} \quad (17.26)$$

$$p_1 V_1 = p_2 V_2 \quad T = \text{constant} \quad (17.28)$$

can be combined into one equation, namely,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad \dots(6-1)$$

Equation **6.1** is a special case of a relation known as the **ideal gas law**.

Hence, we see that the three previous laws, which were developed experimentally, are special cases of this ideal gas law, when either the pressure, volume, or temperature is held constant.

The ideal gas law is a more general equation in that none of the variables must be held constant.

Equation **6.1** expresses the relation between the pressure, volume, and temperature of the gas at one time, with the pressure, volume, and temperature at any other time.

For this equality to hold for any time, it is necessary that:

$$\frac{pV}{T} = \text{constant} \quad \dots(6-2)$$

This constant must depend on the quantity or mass of the gas.

A convenient unit to describe the amount of the gas is the mole.

## Properties of Matter

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*One **mole** of any gas is that amount of the gas that has a mass in grams equal to the atomic or molecular mass (**M**) of the gas.*

The terms atomic mass and molecular mass are often erroneously called atomic weight and molecular weight in chemistry.

As an example of the use of the mole, consider the gas oxygen.

One molecule of oxygen gas consists of two atoms of oxygen, and is denoted by **O<sub>2</sub>**.

The atomic mass of oxygen **16.00**.

The molecular mass of one mole of oxygen gas is therefore

$$M_{O_2} = 2(16) = 32 \text{ g/mole}$$

Thus, one mole of oxygen has a mass of **32 g**.

The mole is a convenient quantity to express the mass of a gas because *one mole of any gas at a temperature of **0 °C** and a pressure of **1 atmosphere**, has a volume of **22.4 liters**.*

*Also Avogadro's law states that every mole of a gas contains the same number of molecules.*

*This number is called **Avogadro's number** **N<sub>A</sub>** and is equal to **6.022 × 10<sup>23</sup>** molecules/mole.*

## Properties of Matter

The mass of any gas will now be represented in terms of the number of moles,  $n$ .

We can write the constant in equation 6.2 as  $n$  times a new constant, which shall be called  $R$ , that is,

$$\frac{pV}{T} = nR \quad \dots(6-3)$$

To determine this constant  $R$  let us evaluate it for 1 mole of gas at a pressure of 1 atm and a temperature of 0 °C, or 273 K, and a volume of 22.4 L.

That is,

$$R = \frac{pV}{nT} = \frac{(1 \text{ atm})(22.4 \text{ L})}{(1 \text{ mole})(273 \text{ K})}$$
$$R = 0.08205 \frac{\text{atm L}}{\text{mole K}}$$

Converted to SI units, this constant is

$$R = \left(0.08205 \frac{\text{L atm}}{\text{mole K}}\right) \left(1.013 \times 10^5 \frac{\text{N/m}^2}{\text{atm}}\right) \left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right)$$
$$R = 8.314 \frac{\text{J}}{\text{mole K}}$$

We call the constant  $R$  the universal gas constant, and it is the same for all gases.

We can now write equation 6.3 as:

$$pV = nRT \quad \dots(6-4)$$

Equation 6.4 is called the **ideal gas equation**.

An ideal gas is one that is described by the ideal gas equation.

*Remember that the temperature  $T$  must always be expressed in Kelvin units.*

### 6.2 The Kinetic Theory of Gases

Up to now the description of a gas has been on the macroscopic level, a large-scale level, where the characteristics of a gas, such as its pressure, volume, and temperature, are measured without regard to the internal structure of the gas itself.

In reality, a gas is composed of a large number of molecules in random motion.

The large-scale characteristics of gases should be explainable in terms of the motion of these molecules.

*The analysis of a gas at this microscopic level (the molecular level) is called the **kinetic theory of gases**.*

In the analysis of a gas at the microscopic level we make the following assumptions:

1. A gas is composed of a very large number of molecules that are in random motion.
2. The volume of the individual molecules is very small compared to the total volume of the gas.
3. The collisions of the molecules with the walls and other molecules are elastic and hence there is no energy lost during a collision.
4. The forces between molecules are negligible except during a collision.

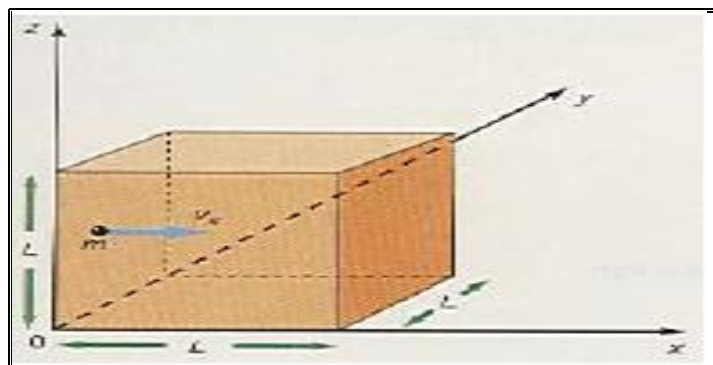
Hence, there is no potential energy associated with any molecule.

5. Finally, we assume that the molecules obey Newton's laws of motion.

## Properties of Matter

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Let us consider one of the very many molecules contained in the box shown in figure 6.1.



**Figure 6.1** The kinetic theory of a gas.

For simplicity we assume that the box is a cube of length  $L$ .

The gas molecule has a mass  $m$  and is moving at a velocity  $v$ .

The  $x$ -component of its velocity is  $v_x$ .

For the moment we only consider the motion in the  $x$ -direction.

The pressure that the gas exerts on the walls of the box is caused by the collision of the gas molecule with the walls.

## Properties of Matter

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The pressure is defined as the force acting per unit area, that is,

$$p = \frac{F}{A} \quad \dots(6-5)$$

where  $A$  is the area of the wall where the collision occurs, and is simply:

$$A = L^2$$

and  $F$  is the force exerted on the wall as the molecule collides with the wall and can be found by Newton's second law in the form

$$F = \frac{\Delta P}{\Delta t} \quad \dots(6-6)$$

*So as not to confuse the symbols for pressure and momentum, we will use the lower case  $p$  for pressure, and we will use the upper case  $P$  for momentum.*

Because momentum is conserved in a collision, the change in momentum of the molecule  $\Delta P$ , is the difference between the momentum after the collision  $P_{AC}$  and the momentum before the collision  $P_{BC}$ .

Also, since the collision is elastic the velocity of the molecule after the collision is  $-v_x$ .



## Properties of Matter

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Therefore, the change in momentum of the molecule is:

$$\begin{aligned}\Delta P &= P_{AC} - P_{BC} = -mv_x - mv_x \\ &= -2mv_x \quad \text{change in momentum of the molecule}\end{aligned}$$

But the change in the momentum imparted to the wall is the negative of this, or:

$$\Delta P = 2mv_x \quad \text{momentum imparted to wall}$$

Therefore, using Newton's second law, the force imparted to the wall becomes:

$$F = \frac{\Delta P}{\Delta t} = \frac{2mv_x}{\Delta t} \quad \dots(6-7)$$

The quantity  $\Delta t$  should be the time that the molecule is in contact with the wall.

But this time is unknown.

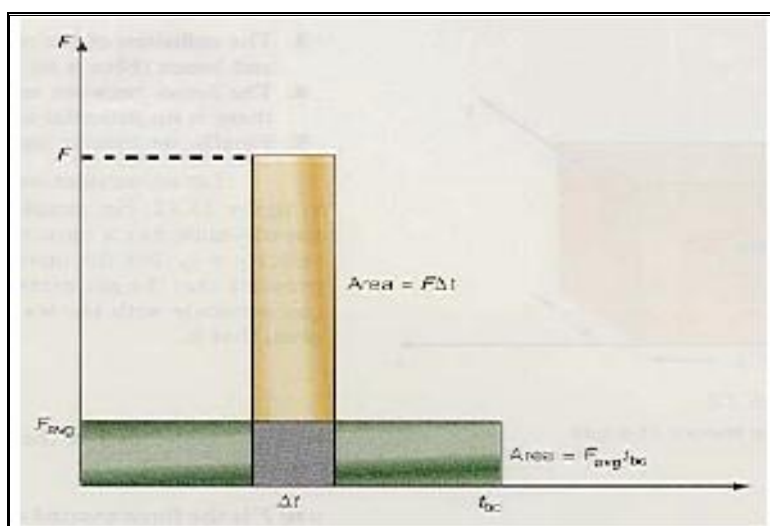
The impulse that the gas particle gives to the wall by the collision is given by:

$$\text{Impulse} = F\Delta t = \Delta P \quad \dots(6-8)$$

and is shown as the area under the force-time graph of figure 6.2.

Because the time  $\Delta t$  for the collision is unknown, a larger time interval  $t_{bc}$ , the time between collisions, can be used with an average force  $F_{avg}$ , such that the product of  $F_{avg}t_{bc}$  is equal to the same impulse as  $F\Delta t$ .

We can see this in figure 6.2.



**Figure 6.2** Since the impulse (the area under the curve) is the same, the change in momentum is the same.

We see that the impulse, which is the area under the curve, is the same in both cases.

At first this may seem strange, but if you think about it, it does make sense.

The actual force in the collision is large, but acts for a very short time.

After the collision, the gas particle rebounds from the first wall, travels back to the far wall, rebounds from it, and then travels to the first wall again, where a new collision occurs.

## Properties of Matter

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For the entire traveling time of the particle the actual force on the wall is zero.

Because we think of the pressure on a wall as being present at all times, it is reasonable to talk about a smaller average force that is acting continuously for the entire time  $t_{bc}$ .

As long as the impulse is the same in both cases, the momentum imparted to the wall is the same in both cases.

Equation 6.7 becomes:

$$\text{Impulse} = F\Delta t = F_{avg}t_{bc} = \Delta P \quad \dots(6-9)$$

The force imparted to the wall, equation 6.6, becomes:

$$F_{avg} = \frac{\Delta P}{t_{bc}} = \frac{2mv_x}{t_{bc}} \quad \dots(6-10)$$

We find the time between the collision  $t_{bc}$  by noting that the particle moves a distance  $2L$  between the collisions.

Since the speed  $v_x$  is the distance traveled per unit time, we have:

$$v_x = \frac{2L}{t_{bc}}$$

Hence, the time between collisions is:

$$t_{bc} = \frac{2L}{v_x} \quad \dots(6-11)$$

## Properties of Matter

Therefore, the force imparted to the wall by this single collision becomes:

$$F_{avg} = \frac{2m v_x}{2L/v_x} = \frac{m v_x^2}{L} \quad \dots(6-12)$$

The total change in momentum per second, and hence the total force on the wall caused by all the molecules is the sum of the forces caused by all of the molecules, that is,

$$F_{avg} = F_{1avg} + F_{2avg} + F_{3avg} + \dots + F_{Navg} \quad \dots(6-13)$$

where  $N$  is the total number of molecules.

Substituting equation 6.12 for each gas molecule, we have:

$$\begin{aligned} F_{avg} &= \frac{m v_{x1}^2}{L} + \frac{m v_{x2}^2}{L} + \frac{m v_{x3}^2}{L} + \dots + \frac{m v_{xN}^2}{L} \\ F_{avg} &= \frac{m(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2)}{L} \end{aligned} \quad \dots(6-14)$$

Let us multiply and divide equation 6.14 by the total number of molecules  $N$ , that is,

$$F_{avg} = \frac{mN(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2)}{L N} \quad \dots(6-15)$$

But the term in parentheses is the definition of an average value.

That is,

$$v_{avg}^2 = \frac{(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2)}{N} \quad \dots(6-16)$$

As an example, if you have four exams in the semester, your average grade is the sum of the four exams divided by 4.

## Properties of Matter

Here, the sum of the squares of the  $x$ -component of the velocity of each molecule, divided by the total number of molecules, is equal to the average of the square of the  $x$ -component of velocity.

Therefore equation 6.14 becomes:

$$F_{avg} = \frac{mN}{L} u_{avg}^2$$

But since the pressure is defined as  $p = F/A$ , from equation 2.2, we have:

$$p = \frac{F_{avg}}{A} = \frac{F_{avg}}{L^2} = \frac{mN}{L^3} u_{avg}^2 = \frac{mN}{V} u_{avg}^2 \quad \dots(6-17)$$

or

$$pV = Nm u_{avg}^2 \quad \dots(6-18)$$

The square of the actual three-dimensional speed is:

$$v^2 = u_x^2 + u_y^2 + u_z^2$$

and averaging over all molecules:

$$u_{avg}^2 = u_{xavg}^2 + u_{yavg}^2 + u_{zavg}^2$$

But because the motion of any gas molecule is random,

$$u_{xavg}^2 = u_{yavg}^2 = u_{zavg}^2$$

That is, there is no reason why the velocity in one direction should be any different than in any other direction, hence their average speeds should be the same.

Therefore,

$$u_{avg}^2 = 3u_{xavg}^2$$

or

$$u_{xavg}^2 = \frac{u_{avg}^2}{3} \quad \dots(6-19)$$

## Properties of Matter

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Substituting equation **6.19** into equation **6.18**, we get:

$$pV = \frac{Nm}{3} v_{\text{avg}}^2$$

Multiplying and dividing the right-hand side by 2, gives:

$$pV = \frac{2}{3} N \left( \frac{mv_{\text{avg}}^2}{2} \right) \quad \dots(6-20)$$

The total number of molecules of the gas is equal to the number of moles of gas times Avogadro's number - the number of molecules in one mole of gas - that is,

$$N = nN_A \quad \dots(6-21)$$

Substituting equation **6.20** into equation **6.19**, gives:

$$pV = \frac{2}{3} nN_A \left( \frac{mv_{\text{avg}}^2}{2} \right) \quad \dots(6-22)$$

Recall that the ideal gas equation was derived from experimental data as:

$$pV = nRT \quad \dots(6-4)$$

The left-hand side of equation **6.4** contains the pressure and volume of the gas, all macroscopic quantities, and all determined experimentally.

The left-hand side of equation **6.22**, on the other hand, contains the pressure and volume of the gas as determined theoretically by Newton's second law.

## Properties of Matter

If the theoretical formulation is to agree with the experimental results, then these two equations must be equal.

Therefore equating equation 6.4 to equation 6.22, we have:

$$nRT = \frac{2}{3} n N_A \left( \frac{m v_{avg}^2}{2} \right)$$

or

$$\frac{3}{2} \left( \frac{R}{N_A} \right) T = \frac{m v_{avg}^2}{2} \quad \dots(6-23)$$

where  $R/N_A$  is the gas constant per molecule.

It appears so often that it is given the special name *the Boltzmann constant* and is designated by the letter  $k$ .

Thus,

$$k = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K} \quad \dots(6-24)$$

Therefore, equation 6.23 becomes:

$$\frac{3}{2} kT = \frac{1}{2} m v_{avg}^2 \quad \dots(6-25)$$

Equation 6.25 relates the macroscopic view of a gas to the microscopic view.

Notice that *the absolute temperature  $T$  of the gas (a macroscopic variable) is a measure of the mean translational kinetic energy of the molecules of the gas (a microscopic variable).*

The higher the temperature of the gas, the greater the average kinetic energy of the gas, the lower the temperature, the smaller the average kinetic energy.

## Properties of Matter

Observe from equation **6.25** that if the absolute temperature of a gas is **0 K**, then the mean kinetic energy of the molecule would be zero and its speed would also be zero.

This was the original concept of absolute zero, a point where all molecular motion would cease.

The average speed of a gas molecule can be determined by solving equation (6.25) for  $v_{\text{avg}}$ .

That is,

$$\frac{1}{2} m v_{\text{avg}}^2 = \frac{3}{2} kT$$

$$v_{\text{avg}}^2 = \frac{3 kT}{m}$$

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

...(6-26)

This particular average value of the speed,  $v_{\text{rms}}$ , is usually called the root-mean-square value, or **rms** value for short, of the speed  $v$ .

It is called the **rms** speed, because it is the square root of the mean of the square of the speed.

Occasionally the **rms** speed of a gas molecule is called the *thermal speed*.

To determine the **rms** speed from equation **6.26**, we must know the mass  $m$  of one molecule.

The mass  $m$  of any molecule is found from

$$m = \frac{M}{N_A}$$

...(6-27)

That is, the mass  $m$  of one molecule is equal to the molecular mass  $M$  of that gas divided by Avogadro's number  $N_A$



### The Language of Physics

#### Lecture 6 ( The ideal gas law )

##### **The ideal gas law**

The general gas law that contains Charles', Gay-Lussac's, and Boyle's law as special cases.

It states that the product of the pressure and volume of a gas divided by the absolute temperature of the gas is a constant .

##### **Mole**

One mole of any gas is that amount of the gas that has a mass in grams equal to the atomic or molecular mass of the gas.

One mole of any gas at a temperature **of 0 °C** and a pressure of one atmosphere, has a volume of **22.4** liters .

##### **Avogadro's number**

Every mole of a gas contains the same number of molecules, namely,  **$6.022 \times 10^{23}$**  molecules.

The mass of one molecule is equal to the molecular mass of that gas divided by Avogadro's number .

##### **Kinetic theory of gases**

The analysis of a gas at the microscopic level, treated by Newton's laws of motion.

The kinetic theory shows that the absolute temperature of a gas is a measure of the mean translational kinetic energy of the molecules of the gas .

Summary of Important Equations

Lecture 6 ( The ideal gas law )

Ideal gas law	$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ $pV = nRT$
Number of molecules	$N = nN_A$
Temperature and mean kinetic energy	$\frac{3}{2} kT = \frac{1}{2} m v_{\text{avg}}^2$
rms speed of a molecule	$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$
Mass of a molecule	$m = \frac{M}{N_A}$
Total mass of the gas	$m_{\text{total}} = nM$

Problems for Lecture

Lecture 6 ( The ideal gas law )

*Example 6.1*

*Find the number of molecules in the gas*

Compute the number of molecules in a gas contained in a volume of **10.0 cm<sup>3</sup>** at a pressure of  **$1.013 \times 10^5$  N/m<sup>2</sup>**, and a temperature of **300 K**.

**Answer :** (  $2.45 \times 10^{20}$  molecules )

*Example 6.2*

*The kinetic energy of a gas molecule*

What is the average kinetic energy of the oxygen and nitrogen molecules in a room at room temperature?

**Answer :** (  $6.07 \times 10^{-21}$  J )

**Example 6.3**

*The rms speed of a gas molecule*

Find the **rms** speed of an oxygen and nitrogen molecule at room temperature?

**Answer :** ( 478 m/s , 511 m/s )