Kirkuk University

Science College

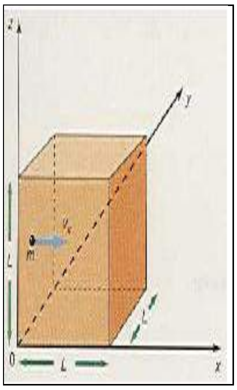
Physics Department

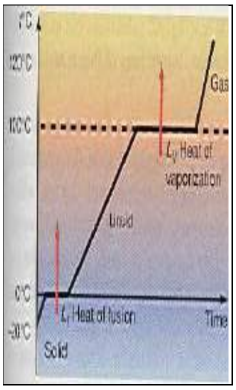
Lectures of

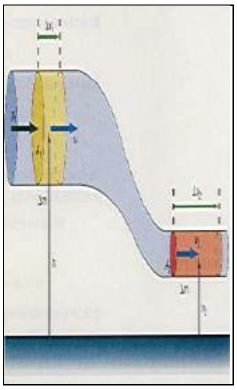
Properties of Matter

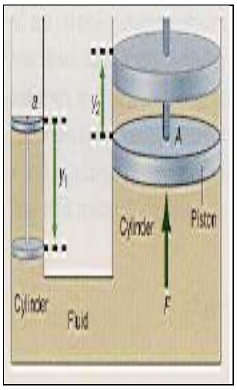
Lecture 5

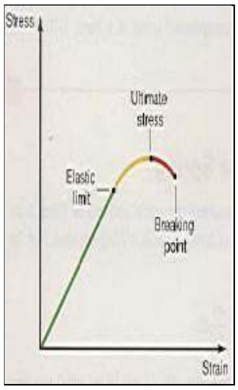
Thermal expansion and gas laws











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5.1 Linear Expansion of Solids

It is a well-known fact that most materials expand when heated.

This expansion is called **thermal expansion.**

If a long thin rod of length L_0, at an initial temperature t_i, is heated to a final temperature t_f, then the rod expands by a small length ΔL, as shown in figure **5.1**.

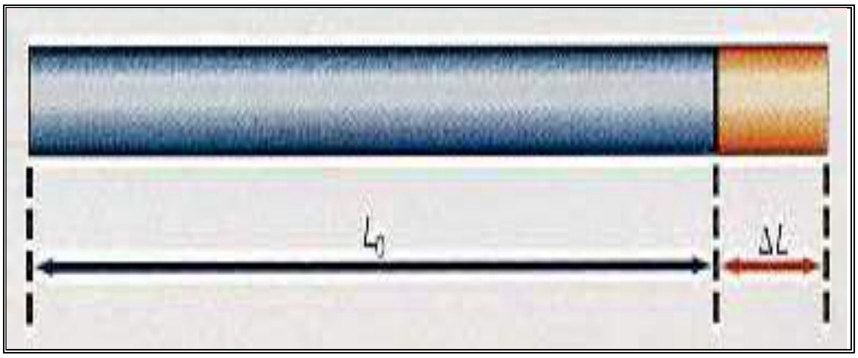

Figure 5.1 **Linear expansion.**

It is found by experiment that the change in length ΔL depends on the temperature change, $\Delta t = t_f - t_i$; the initial length of the rod L_0; and a constant that is characteristic of the material being heated.

The experimentally observed linearity between ΔL and $L_0 \Delta t$ can be represented by the equation:

$$\Delta L = \alpha L_{\circ} \Delta t \ldots (5-1)$$

We call the constant α the *coefficient of linear expansion;* table **5.1** gives this value for various materials.

Table 5.1

Coefficients of Thermal Expansion

Table (5 – 1)

Coefficients of Thermal Expansion		
Material	α Coefficient of Linear Expansion $\times 10^{-5}$ /^{0}C	β Coefficient of Volume Expansion $\times 10^{-4}$ /^{0}C
Aluminum	2.4	
Brass	1.8	
Copper	1.7	
Iron	1.2	
Lead	3.0	
Steel	1.2	
Zinc	2.6	
Glass (ordinary)	0.9	
Glass (Pyrex)	0.32	
Ethyl alcohol		11.0
Water		2.1
Mercury		1.8
Glass (Pyrex)		0.096
All noncondensing gases at constant pressure and 0 ^{0}C.		36.6

The change in length is rather small, but it is, nonetheless, very significant.

The expansion of the solid can be explained by looking at the molecular structure of the solid.

The molecules of the substance are in a lattice structure.

Any one molecule is in equilibrium with its neighbors, but vibrates about that equilibrium position.

As the temperature of the solid is increased, the vibration of the molecule increases.

However, the vibration is not symmetrical about the original equilibrium position.

As the temperature increases the equilibrium position is displaced from the original equilibrium position.

Hence, the mean displacement of the molecule from the original equilibrium position also increases, thereby spacing all the molecules farther apart than they were at the lower temperature.

The fact that all the molecules are farther apart manifests itself as an increase in length of the material.

Hence, linear expansion can be explained as a molecular phenomenon.

The large force associated with the expansion comes from the large molecular forces between the molecules.

5.2 Area Expansion of Solids

For the long thin rod of section **5.1**, only the length change was significant and that was all that we considered.

But solids expand in all directions.

If a rectangle of thin material of length L_1 and width L_2, at an initial temperature of t_i, is heated to a new temperature t_f, the rectangle of material expands, as shown in figure **5.2**.

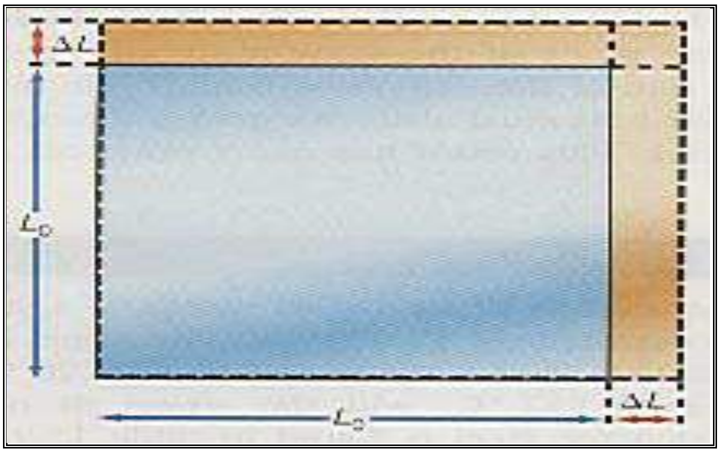

Figure 5.2 **Expansion in area.**

The original area of the rectangle is given by:

$$\boxed{A = L_1 L_2 \ldots (5-2)}$$

The change in area of the rectangle caused by a change in temperature of the material is found by differentiating equation **5.4** with respect to the temperature t.

That is:

$$\frac{dA}{dt} = L_1\frac{dL_2}{dt} + L_2\frac{dL_1}{dt}$$

Let us now divide both sides of this equation by equation **5.2** to get:

$$\frac{1}{A}\frac{dA}{dt} = \frac{L_1}{L_1L_2}\frac{dL_2}{dt} + \frac{L_2}{L_1L_2}\frac{dL_1}{dt}$$

which yields

$$\frac{1}{A}\frac{dA}{dt} = \frac{1}{L_2}\frac{dL_2}{dt} + \frac{1}{L_1}\frac{dL_1}{dt}$$

But :

$$\frac{1}{L_2}\frac{dL_2}{dt} = \alpha = \frac{1}{L_1}\frac{dL_1}{dt}$$

Hence

$$\frac{1}{A}\frac{dA}{dt} = \alpha + \alpha = 2\alpha$$

The change in area ***dA*** of a material, caused by a change in temperature ***dt***, is:

$$\boxed{dA = 2\alpha A dt ...(5-3)}$$

Equation **5.3** gives us the area expansion ***dA*** of a material of original area ***A*** when subjected to a temperature change ***dt***.

Note that the coefficient of area expansion is twice the coefficient of linear expansion.

Although we have derived this result for a rectangle it is perfectly general and applies to any area.

For example, if the material was circular in shape, the original area A_0 would be computed from the area of a circle of radius r_0 as:

$$A_0 = \pi r_0^2$$

We would then find the change in area from equation **5.5**.

The new area can be found by adding the change in area ΔA to the original area A_0 as:

$$\boxed{A = A_\circ + \Delta A ..(5-4)}$$

5.3 Volume Expansion of Solids and Liquids

All materials have three dimensions, length, width, and height.

When a body is heated, all three dimensions should expand and hence its volume should increase.

Let us consider a solid box of length L_1, width L_2, and height L_3, at an initial temperature t_i.

If the material is heated to a new temperature t_f, then each side of the box undergoes an expansion dL.

The volume of the solid box is given by:

$$\boxed{V = L_1 L_2 L_3 ...(5-5)}$$

The change in volume of the box caused by a change in temperature of the material is found by differentiating equation **5.5** with respect to the temperature ***t***.

That is:

$$\frac{dV}{dt} = L_2L_3\frac{dL_1}{dt} + L_3L_1\frac{dL_2}{dt} + L_1L_2\frac{dL_3}{dt}$$

Let us now divide both sides of this equation by equation **5.5** to get:

$$\frac{1}{V}\frac{dV}{dt} = \frac{L_2L_3}{L_1L_2L_3}\frac{dL_1}{dt} + \frac{L_3L_1}{L_1L_2L_3}\frac{dL_2}{dt} + \frac{L_1L_2}{L_1L_2L_3}\frac{dL_3}{dt}$$

which yields:

$$\frac{1}{V}\frac{dV}{dt} = \frac{1}{L_1}\frac{dL_1}{dt} + \frac{1}{L_2}\frac{dL_2}{dt} + \frac{1}{L_3}\frac{dL_3}{dt}$$

But

$$\frac{1}{L_1}\frac{dL_1}{dt} = \frac{1}{L_2}\frac{dL_2}{dt} = \frac{1}{L_3}\frac{dL_3}{dt} = \alpha$$

Hence

$$\frac{1}{V}\frac{dV}{dt} = \alpha + \alpha + \alpha = 3\alpha$$

The change in area ***dV*** of a material, caused by a change in temperature ***dt***, is:

$$\boxed{dV = 3\alpha V dt \ldots(5-6)}$$

Equation **5.6** gives us the volume expansion ***dV*** of a material of original volume ***V*** when subjected to a temperature change ***dt***.

Note that the coefficient of volume expansion is three times the coefficient of linear expansion.

5.4 Volume Expansion of Gases: Charles' Law

Consider a gas placed in a tank, as shown in figure **5.3**.

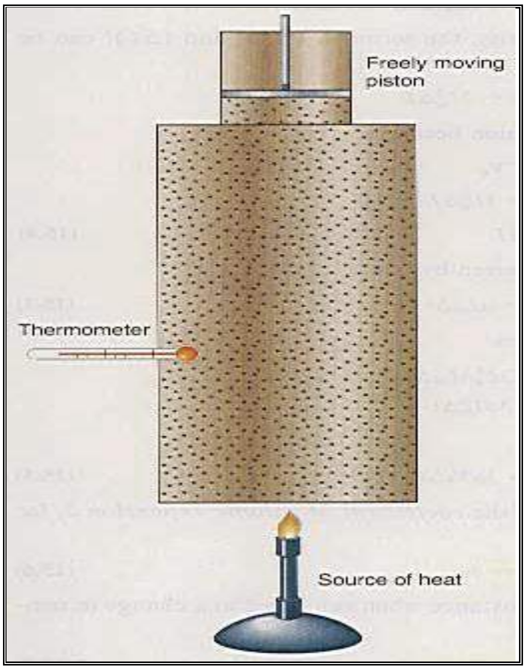

Figure 5.3 Volume expansion of a gas.

The weight of the piston exerts a constant pressure on the gas.

When the tank is heated, the pressure of the gas first increases.

But the increased pressure in the tank pushes against the freely moving piston, and the piston moves until the pressure inside the tank is the same as the pressure exerted by the weight of the piston.

Therefore the pressure in the tank remains a constant throughout the entire heating process.

The volume of the gas increases during the heating process, as we can see by the new volume occupied by the gas in the top cylinder.

In fact, we find the increased volume by multiplying the area of the cylinder by the distance the piston moves in the cylinder.

If the volume of the gas is plotted against the temperature of the gas, in Celsius degrees, we obtain the straight line graph in figure **5.4**.

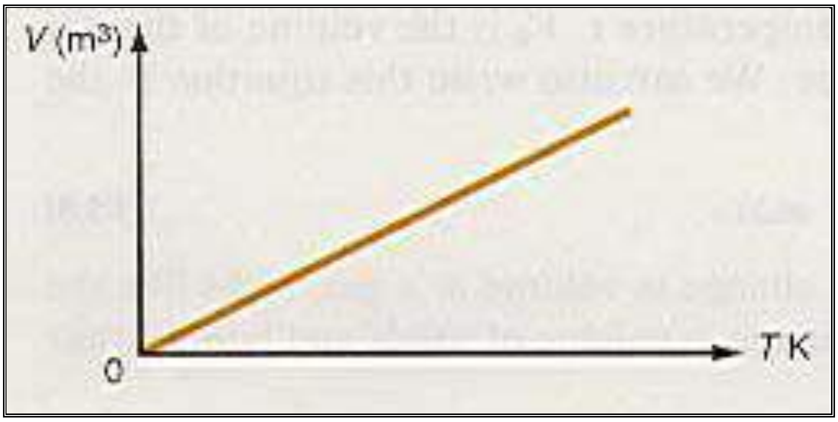

***Figure 5.4* The volume *V* of a gas is directly proportional to its absolute temperature *T*.**

Thus, *the volume of a gas at constant pressure is directly proportional to the absolute temperature of the gas.*

This result is known as ***Charles' law.***

Therefore,

$$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2} \ldots (5-7)}$$

which is form of Charles' law.

5.5 Gay-Lussac's Law

Consider a gas contained in a tank, as shown in figure **5.5**.

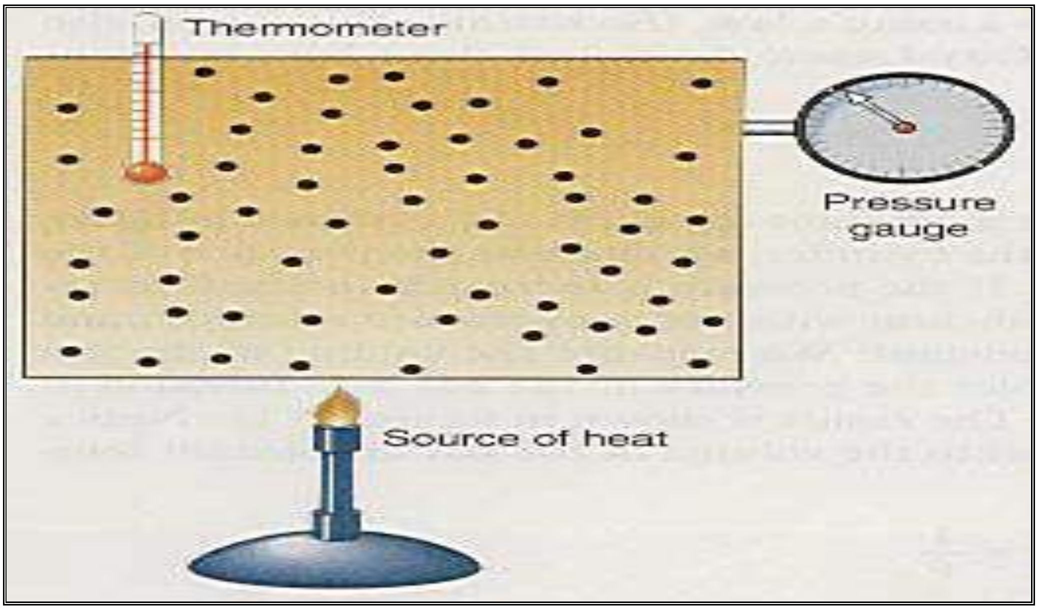

Figure 5.5 **Changing the pressure of a gas.**

The tank is made of steel and there is a negligible change in the volume of the tank, and hence the gas, as it is heated.

A pressure gauge attached directly to the tank, is calibrated to read the absolute pressure of the gas in the tank.

A thermometer reads the temperature of the gas in degrees Celsius.

The tank is heated, thereby increasing the temperature and the pressure of the gas, which are then recorded.

If we plot the pressure of the gas versus the temperature, we obtain the graph of figure **5.6**.

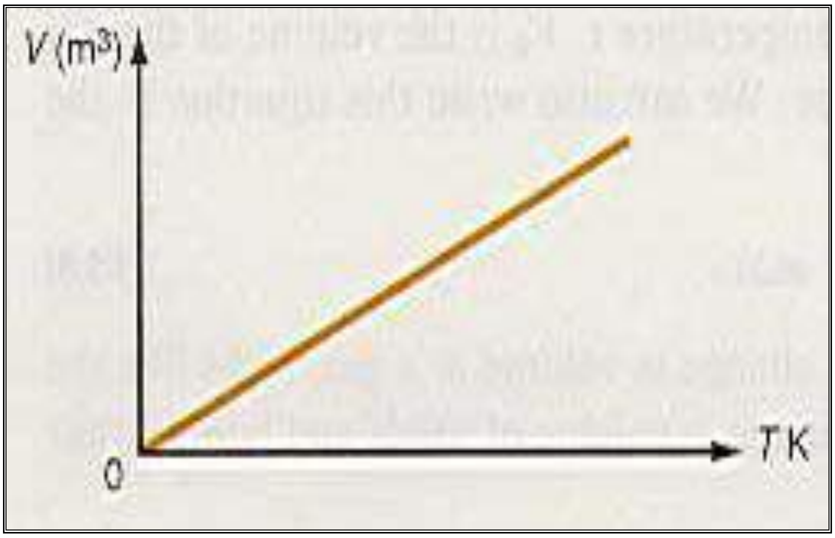

Figure 5.6 **A plot of pressure versus temperature for a gas.**

The absolute pressure of a gas at constant volume is directly proportional to the absolute temperature of the gas, a result known as ***Gay-Lussac's law,*** in honor of the French chemist Joseph Gay-Lussac (1778-1850).

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \ldots (5-8)$$

Equation **5.8** is A form of Gay-Lussac's law. (Sometimes this law is also called Charles' law, since Charles and Gay-Lussac developed these laws independently of each other.)

5.6 Boyle's Law

Consider a gas contained in a cylinder at a constant temperature, as shown in figure **5.7**.

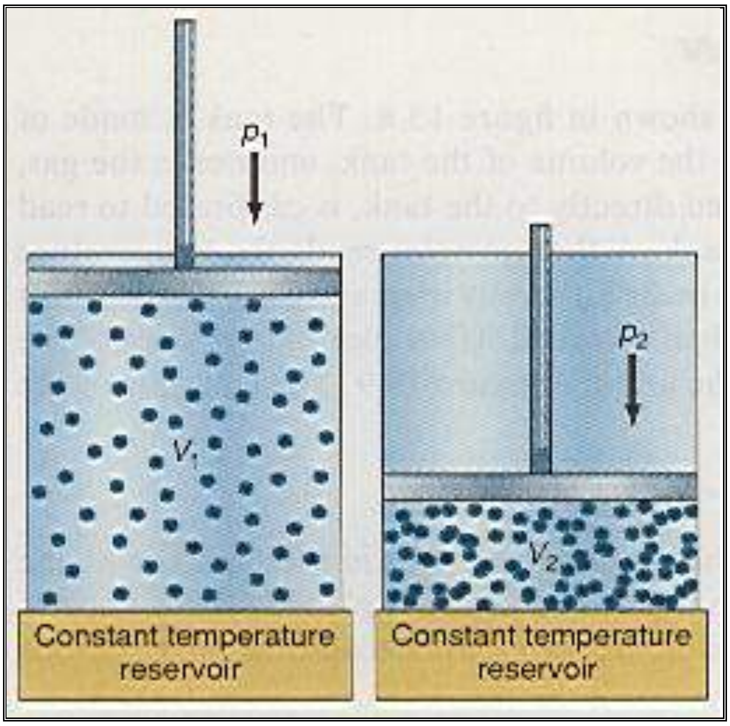

Figure 5.7 **The change in pressure and volume of a gas at constant temperature.**

By pushing the piston down into the cylinder, we increase the pressure of the gas and decrease the volume of the gas.

If the pressure is increased in small increments, the gas remains in thermal equilibrium with the temperature reservoir, and the temperature of the gas remains a constant.

We measure the volume of the gas for each increase in pressure and then plot the pressure of the gas as a function of the reciprocal of the volume of the gas.

The result is shown in figure **5.8**.

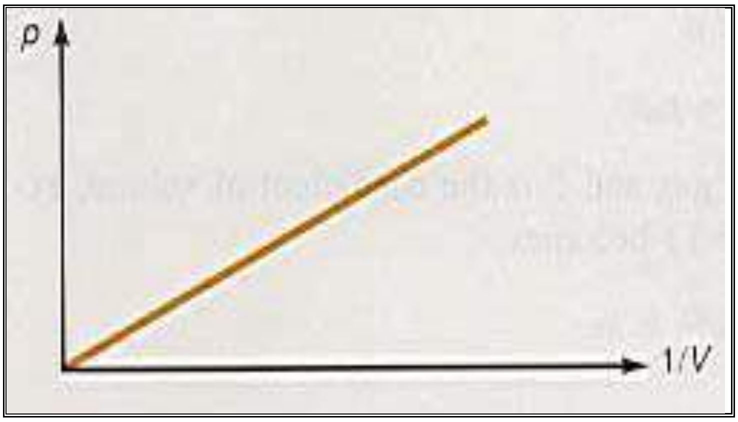

Figure 5.8 **Plot of the pressure *p* versus the reciprocal of the volume 1/*V* for a gas.**

Notice that the pressure is inversely proportional to the volume of the gas at constant temperature.

We can write this as: $p \propto \frac{1}{V}$

Or

$$\boxed{PV = constant ...(5-9)}$$

That is, the product of the pressure and volume of a gas at constant temperature is equal to a constant, a result known as Boyle's law, in honor of the British physicist and chemist Robert Boyle (**1627-1691**).

For a gas in two different equilibrium states at the same temperature, we write this as:

$$p_1V_1 = \text{constant}$$

and

$$p_2V_2 = \text{constant}$$

Therefore,

$$p_1V_1 = p_2V_2 \qquad T = \text{constant}$$

another form of Boyle's law.

The Language of Physics

Lecture 5 (Thermal Expansion and the Gas Laws)

Thermal expansion

Most materials expand when heated .

Charles' law

The volume of a gas at constant pressure is directly proportional to the absolute temperature of the gas .

Gay-Lussac's law

The absolute pressure of a gas at constant volume is directly proportional to the absolute temperature of the gas .

Boyle's law

The product of the pressure and volume of a gas at constant temperature is equal to a constant .

Summary of Important Equations

Lecture 5 (Thermal Expansion and the Gas Laws)

Linear expansion	$\Delta L = \alpha L_0 \Delta t$
Area expansion	$dA = 2\alpha A dt$
Volume expansion	$\Delta V = 3\alpha V_0 \Delta t$
Charles' law	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ p = constant
Gay-Lussac's law	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$ V = constant
Boyle's law	$p_1 V_1 = p_2 V_2$ T = constant

Problems for Lecture

Lecture 5 (Thermal Expansion and the Gas Laws)

Example 5.1

Expansion of a railroad track

A steel railroad track was **30.0 m** long when it was initially laid at a temperature of **−6.70 ^{0}C**. What is the change in length of the track when the temperature rises to **35.0 ^{0}C**? What is the new length of the track ?

Answer : (0.0150 m) , (30.0150 m)

Example 5.2

The change in area

An aluminum sheet **2.50 m** long and **3.24 m** wide is connected to some posts when it was at a temperature of **−10.5 ^{0}C**. What is the change in area of the aluminum sheet when the temperature rises to **65.0 ^{0}C**? What is the new area of the sheet ?

Answer : (0.0294 m^2) , (8.13 m^2)

Example 5.3

The change in volume.

An aluminum box **0.750 m** long, **0.250 m** wide, and **0.450 m** high is at a temperature of **−15.6 ^{0}C**. What is the change in volume of the aluminum box when the temperature rises to **120 ^{0}C**? What is the new length of the track ?

Answer : (0.00082 m^3) , (0.0852 m^3)

Example 5.4

Find the temperature of the gas

The pressure of an ideal gas is kept constant while **3.00 m^3** of the gas, at an initial temperature of **50.0 ^{0}C**, is expanded to **6.00 m^3**. What is the final temperature of the gas?

Answer : (646 K)

Example 5.5

Find the volume of the gas

A balloon is filled with helium at a pressure of **2.03×10^5 N/m^2**, a temperature of **35.0 ^{0}C**, and occupies a volume of **3.00 m^3**. The balloon rises in the atmosphere. When it reaches a height where the pressure is **5.07×10^4 N/m^2**, and the temperature is **−20.0 ^{0}C**, what is its volume?

Answer : (9.87 m^3)

Example 5.6

Find the pressure of the gas

What is the pressure produced by **2.00 moles** of a gas at **35.0 ^{0}C** contained in a volume of **5.00×10^{-3} m^3**?

Answer : (1.02 x 10^6 N/m^2)