Kirkuk University

Science College

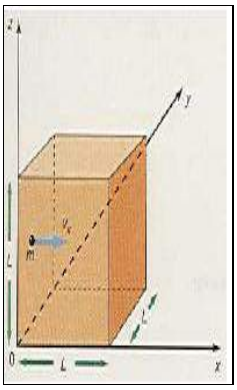
Physics Department

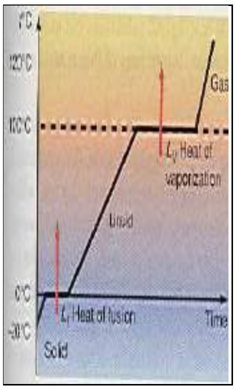
Lectures of

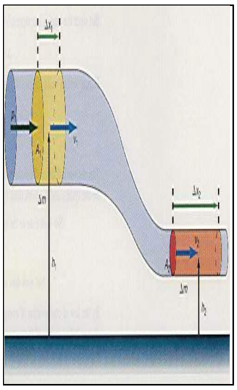
Properties of Matter

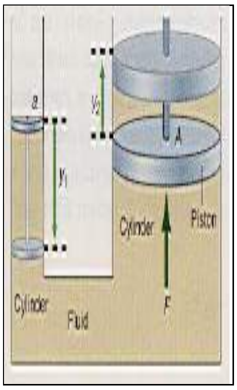
Lecture =3=

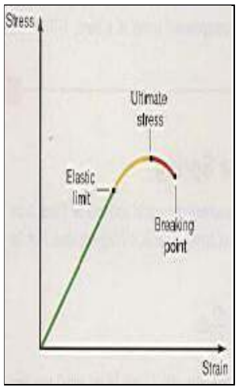
Dynamic Fluids











Assistant professor Dr.Jawdet Hedayet Mohammed

Lecturer in Kirkuk University

Science College – Physics Department

3.1 The Equation of Continuity

In the Lecture **2** , we have studied only fluids at rest.

Let us now study fluids in motion, the subject matter of hydrodynamics.

The study of fluids in motion is relatively complicated, but the analysis can be simplified by making a few assumptions.

Let us assume that the fluid is incompressible and flows freely without any turbulence or friction between the various parts of the fluid itself and any boundary containing the fluid, such as the walls of a pipe.

A fluid in which friction can be neglected is called a ***nonviscous fluid***.

A fluid, flowing steadily without turbulence, is usually referred to as being in ***streamline flow***.

The rather complicated analysis is further simplified by the use of two great conservation principles: the conservation of mass, and the conservation of energy.

The law of conservation of mass results in a mathematical equation, usually called the equation of continuity.

The law of conservation of energy is the basis of Bernoulli's theorem, the subject matter of section **(3.2)**.

Let us consider an ***incompressible fluid*** flowing in the pipe of figure **(3.1)**.

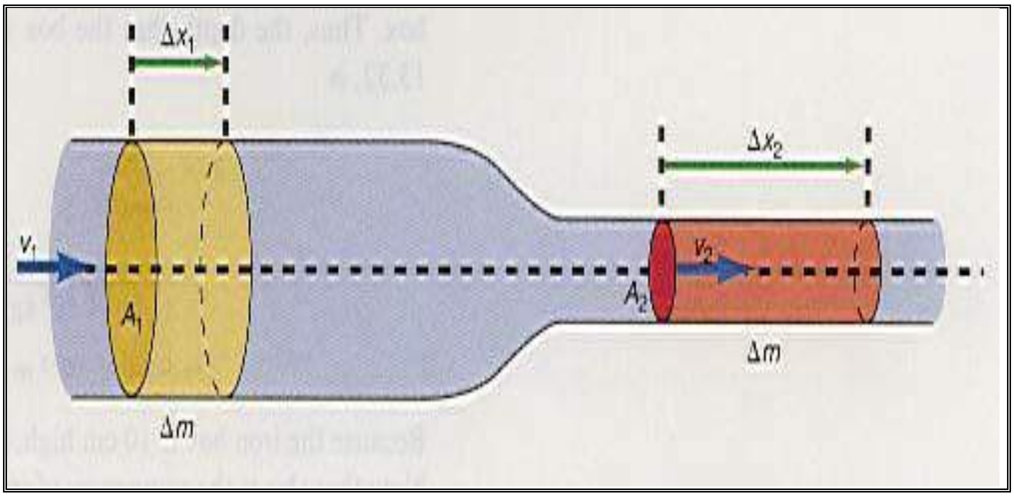


Figure (3.1) **: The law of conservation of mass and the equation of continuity.**

At a particular instant of time the small mass of fluid **(Δm)** , shown in the left-hand portion of the pipe will be considered.

This mass is given by a slight modification of equation **(3.1)**, as:

$$\Delta m = \rho \Delta V (lefthand) ...(3-1)$$

Because the pipe is cylindrical, the small portion of volume of fluid is given by the product of the cross-sectional area **(A_1)** times the length of the pipe **(Δx_1)** containing the mass **(Δm)** , that is:

$$\Delta V = A_1 \Delta x_1 ...(3-2)$$

The length **(Δx_1)** of the fluid in the pipe is related to the velocity **(v_1)** of the fluid in the left-hand pipe.

Because the fluid in **(Δx_1)** moves a distance **(Δx_1)** in time **(Δt)** .

Thus:

$$\Delta x_1 = v_1 \Delta t ...(3-3)$$

Substituting equation **(3.3)** into equation **(3.2)**, we get for the volume of fluid:

$$\boxed{\Delta V = A_1 v_1 \Delta t ...(3-4)}$$

Substituting equation **(3.4)** into equation **(3.1)** yields the mass of the fluid as :

$$\boxed{\Delta m = \rho A_1 v_1 \Delta t ...(3-5)}$$

We can also express this as the rate at which the mass is flowing in the left-hand portion of the pipe by dividing both sides of equation **(3.5)** by **(Δt)** , thus :

$$\boxed{\frac{\Delta m}{\Delta t} = \rho A_1 v_1 (lefthand)...(3-6)}$$

When this fluid reaches the narrow constricted portion of the pipe to the right in figure **(3.1)**, the same amount of mass **(Δm)** is given by :

$$\boxed{\Delta m = \rho \Delta V (righthand)...(3-7)}$$

But since **(ρ)** is a constant, the same mass **(Δm)** must occupy the same volume **(ΔV)** .

However, the right-hand pipe is constricted to the narrow cross-sectional area **(A_2)** .

Thus, the length of the pipe holding this same volume must increase to a larger value **(Δx_2)**, as shown in figure **(3.1)**.

Hence, the volume of fluid is given by :

$$\boxed{\Delta V = A_2 \Delta x_2 ...(3.8)}$$

The length of pipe **(Δx_2)** occupied by the fluid is related to the velocity of the fluid by:

$$\boxed{\Delta x_2 = v_2 \Delta t ...(3-9)}$$

Substituting equation **(3.9)** back into equation **(3.8)** , we get for the volume of fluid:

$$\boxed{\Delta V = A_2 v_2 \Delta t ...(3-10)}$$

It is immediately obvious that since ***(A₂)*** has decreased, v_2 must have increased for the same volume of fluid to flow.

Substituting equation **(3.10)** back into equation **(3.7)** , the mass of the fluid flowing in the right-hand portion of the pipe becomes:

$$\boxed{\Delta m = \rho A_2 v_2 \Delta t ...(3-11)}$$

Dividing both sides of equation **(3.11)** by **(Δ*t*)** yields the rate at which the mass of fluid flows through the right-hand side of the pipe, that is:

$$\boxed{\frac{\Delta m}{\Delta t} = \rho A_2 v_2 \,(righthand)...(3-12)}$$

But *the **law of conservation of mass** states that mass is neither created nor destroyed in any ordinary mechanical or chemical process.*

Hence, the law of conservation of mass can be written as :

Mass flowing into the pipe = mass flowing out of the pipe

or

$$\boxed{\frac{\Delta m}{\Delta t}(lefthand) = \frac{\Delta m}{\Delta t}(righthand)...(3-13)}$$

Thus, setting equation **(3.6)** equal to equation **(3.12)** yields:

$$\boxed{\rho A_1 v_1 = \rho A_2 v_2 ...(3-14)}$$

Equation **(3.14)** is called **the equation of continuity** and is an indirect statement of the law of conservation of mass.

Since we have assumed an incompressible fluid, the densities on both sides of equation **(3.14)** are equal and can be canceled out leaving :

$$\boxed{A_1 v_1 = A_2 v_2 ...(3.15)}$$

*Equation **(3.15)** is a special form of the equation of continuity for incompressible fluids (i.e., liquids).*

Applying equation **(3.15)** to figure **(3.1)**, we see that the velocity of the fluid **(v_2)** in the narrow pipe to the right is given by :

$$\boxed{v_2 = \frac{A_1}{A_2} v_1 ...(3-16)}$$

Because the cross-sectional area **(A_1)** is greater than the cross-sectional area **(A_2)** , the ratio **(A_1/A_2)** is greater than one and thus the velocity **(v_2)** must be greater than **(v_1)** .

*Therefore, as a general rule, the equation of continuity for liquids, equation **(3.15)** , says that when the cross-sectional area of a pipe gets smaller, the velocity of the fluid must become greater in order that the same amount of mass passes a given point in a given time.*

Conversely, when the cross-sectional area increases, the velocity of the fluid must decrease.

Equation **(3.15)** , the equation of continuity, is sometimes written in the equivalent form :

$$\boxed{Av = constant ...(3.17)}$$

3.2 Bernoulli's Theorem

Bernoulli's theorem is a fundamental theory of hydrodynamics that describes a fluid in motion.

It is really the application of the law of conservation of energy to fluid flow.

Let us consider the fluid flowing in the pipe of figure **(3.2).**

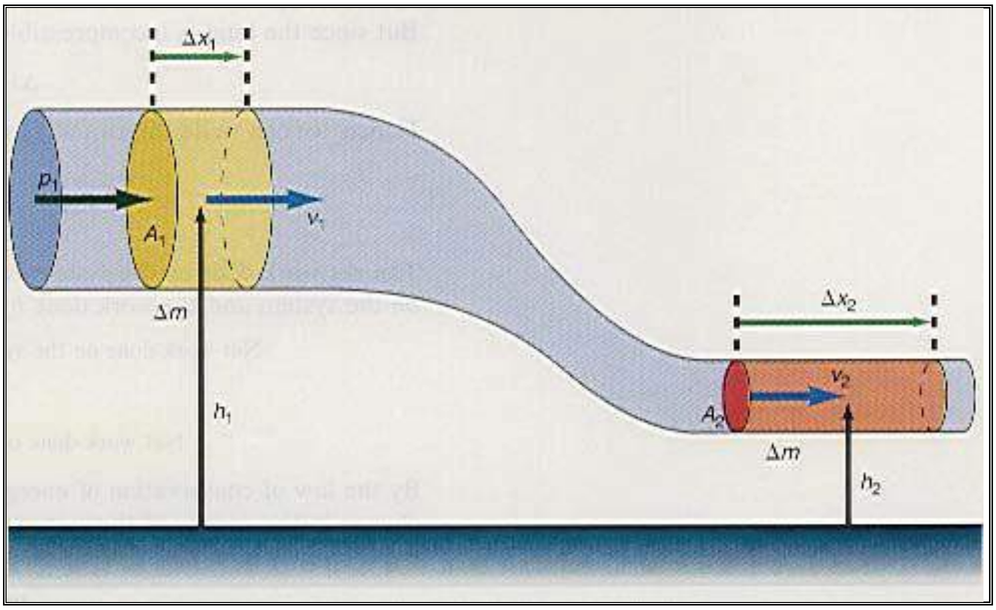


Figure (3.2) **: Bernoulli's theorem.**

The left-hand side of the pipe has a uniform cross-sectional area **(A_1)**, which eventually tapers to the uniform cross-sectional area **(A_2)** of the right-hand side of the pipe.

The pipe is filled with a nonviscous, incompressible fluid.

A uniform pressure **(p_1)** is applied, such as from a piston, to a small element of mass of the fluid **(Δm)** and causes this mass to move through a distance **(Δx_1)** of the pipe.

Because the fluid is incompressible, the fluid moves throughout the rest of the pipe.

The same small mass **(Δm)** , at the right-hand side of the pipe, moves through a distance **(Δx_2)** .

The work done on the system by moving the small mass through the distance **(Δx_1)** is given by the definition of work as :

$$W_1 = F_1 \Delta x_1$$

We can express the force **(F_1)** moving the mass to the right in terms of the pressure exerted on the fluid as :

$$F_1 = P_1 A_1$$

Hence,

$$W_1 = P_1 A_1 \Delta x_1$$

But

$$A_1 \Delta x_1 = \Delta V_1$$

the volume of the fluid moved through the pipe.

Thus, we can write the work done on the system as :

$$\boxed{W_1 = P_1 \Delta V_1 (Work done on the System) ...(3-18)}$$

As this fluid moves through the system, the fluid itself does work by exerting a force **(F_2)** on the mass **(Δm)** on the right side, moving it through the distance **(Δx_2)** .

Hence, the work done by the fluid system is :

$$W_2 = F_2 \Delta x_2$$

But we can express the force **(F_2)** in terms of the pressure **(p_2)** on the right side by :

$$F_2 = P_2 A_2$$

Therefore, the work done by the system is :

$$W_2 = P_2 A_2 \Delta x_2$$

But

$$A_2\Delta x_2 = \Delta V_2$$

the volume moved through the right side of the pipe.

Thus, the work done by the system becomes :

$$\boxed{W_2 = P_2\Delta V_2 (Work done by the System) ...(3-19)}$$

But since the fluid is incompressible,

$$\Delta V_1 = \Delta V_2 = \Delta V$$

Hence, we can write the two work terms, equations **(3.18)** and **(3.19)**, as :

$$W_1 = p_1\Delta V$$
$$W_2 = p_2\Delta V$$

The net work done on the system is equal to the difference between the work done *on* the system and the work done *by* the system.

Hence:

$$Net work done on the system = W_{on} - W_{by}$$

$$= W_1 - W_2 = P_1\Delta V - P_2\Delta V$$

$$\boxed{Net work done on the system = (P_1 - P_2).\Delta V ...(3-20)}$$

By the law of conservation of energy, the net work done on the system produces a change in the energy of the system.

The fluid at position **(1)** is at a height **(h_1)** above the reference level and therefore possesses a potential energy given by:

$$\boxed{PE_1 = (\Delta m).gh_1 ...(3-21)}$$

Because this same fluid is in motion at a velocity **(v_1)** , it possesses a kinetic energy given by:

$$\boxed{KE_1 = \frac{1}{2}.(\Delta m).v_1^{\,2} ...(3-22)}$$

Similarly at position **(2)**, the fluid possesses the potential energy :

$$PE_2 = (\Delta m).gh_2 ...(3-23)$$

and the kinetic energy :

$$KE_2 = \frac{1}{2}.(\Delta m).v_2{}^2 ...(3-24)$$

Therefore, we can now write the law of conservation of energy as :

$$NetWorkDoneOnTheSystem = ChangeInEnergyOfTheSystem...(3-25)$$

$$NetWorkDoneOnTheSystem = (E_{tot})_2 - (E_{tot})_1 ...(3-26)$$

$$NetWorkDoneOnTheSystem = (PE_2 + KE_2) - (PE_1 + KE_1)...(3-27)$$

Substituting equations **(3.20)** into equation **(3.27)** we get :

$$(P_1 - P_2)\Delta V = \left[(\Delta m)gh_2 + \frac{1}{2}(\Delta m)v_2{}^2\right] - \left[(\Delta m)gh_1 + \frac{1}{2}(\Delta m)v_1{}^2\right] ...(3-28)$$

But the total mass of fluid moved **(Δm)** is given by :

$$\Delta m = \rho \Delta V ...(3-29)$$

Substituting equation **(3.29**) back into equation **(3.28)**, gives :

$$(P_1 - P_2)\Delta V = \rho(\Delta V)gh_2 + \frac{1}{2}\rho(\Delta V)v_2{}^2 - \rho(\Delta V)gh_1 - \frac{1}{2}\rho(\Delta V)v_1{}^2$$

Dividing each term by **(ΔV)** gives :

$$(P_1 - P_2) = \rho gh_2 + \frac{1}{2}\rho v_2{}^2 - \rho gh_1 - \frac{1}{2}\rho v_1{}^2 ...(3-30)$$

If we place all the terms associated with the fluid at position **(1)** on the left-hand side of the equation and all the terms associated with the fluid at position **(2)** on the right-hand side, we obtain :

$$P_1 + \rho g h_1 + \frac{1}{2}\rho {v_1}^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho {v_2}^2 \ldots(3-31)$$

Equation **(3.31)** is the mathematical statement of :

Bernoulli's theorem:

It says that the sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume at any one location of the fluid is equal to the sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume at any other location in the fluid, for a ***nonviscous, incompressible fluid*** *in* ***streamlined flow****.*

Since this sum is the same at any arbitrary point in the fluid, the sum itself must therefore be a constant.

Thus, we sometimes write Bernoulli's equation in the equivalent form:

$$P + \rho g h + \frac{1}{2}\rho v^2 = cons\tan t \ldots(3-32)$$

3.3 Application of Bernoulli's Theorem

Let us now consider some special cases of Bernoulli's theorem.

3.3.1 The Venturi Meter

Let us first consider the constricted tube studied in figure **(3.1)** and slightly modified and redrawn in figure **(3.3(a)).**

Since the tube is completely horizontal **($h_1 = h_2$)** and there is no difference in potential energy between the locations **(1)** and **(2)** .

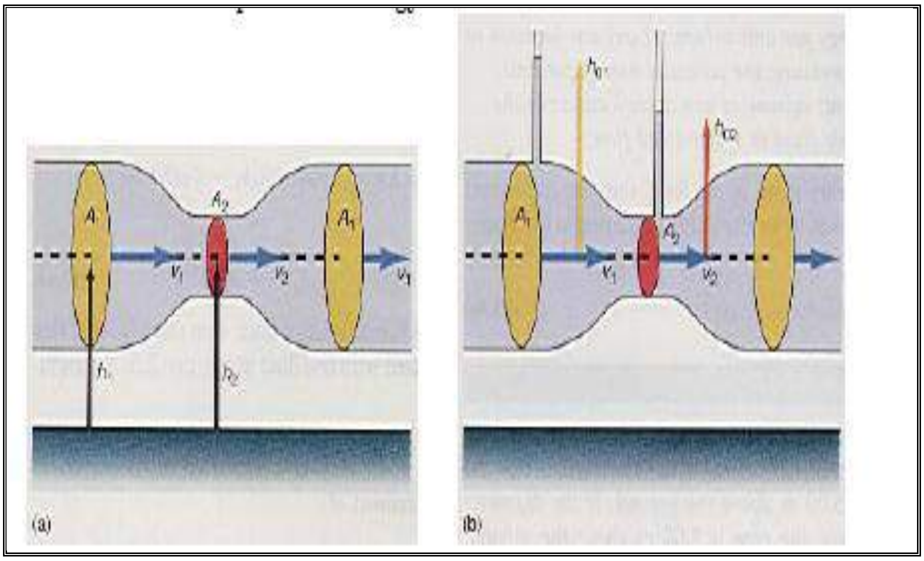


Figure (3.3) **: A Venturi meter.**

Bernoulli's equation therefore reduces to :

$$\boxed{P_1 + \frac{1}{2}\rho v_1^{\,2} = P_2 + \frac{1}{2}\rho v_2^{\,2} \ldots (3-33)}$$

But by the equation of continuity,

$$v_2 = \frac{A_1}{A_2}.v_1 \ldots (3-16)$$

Since **(*A*₁)** is greater than ***(A₂)*** , ***(v₂)*** must be greater than ***(v₁)*** ,as shown before.

Let us rewrite equation **(3.33)** as :

$$p_2 = p_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

or

$$\boxed{P_2 = P_1 + \frac{1}{2}\rho(v_1^{\ 2} - v_2^{\ 2}) \ldots (3-34)}$$

But since **(v_2)** is greater than $\boldsymbol{v_1}$, the quantity **(1/2)ρ($v_1^2 - v_2^2$)** is a negative quantity and when we subtract it from **(p_1)** , **(p_2)** must be less than **(p_1)** .

Thus, not only does the fluid speed up in the constricted tube, but the pressure in the constricted tube also decreases.

*The effect of the decrease in pressure with the increase in speed of the fluid in a horizontal pipe is called the **Venturi effect***, and a simple device called a **Venturi meter,** based on this Venturi effect, is used to measure the velocity of fluids in pipes.

A Venturi meter is shown schematically in figure **(3.3(b)).**

The device is basically the same as the pipe in **(3.3(a))** except for the two vertical pipes connected to the main pipe as shown.

These open vertical pipes allow some of the water in the pipe to flow upward into the vertical pipes.

The height that the water rises in the vertical pipes is a function of the pressure in the horizontal pipe.

As just seen, the pressure in pipe **(1)** is greater than in pipe **(2)** and thus the height of the vertical column of water in pipe **(1)** will be greater than the height in pipe **(2)**.

By actually measuring the height of the fluid in the vertical columns the pressure in the horizontal pipe can be determined by the hydrostatic equation .

Thus, the pressure in pipe **(1)** is:

$$p_1 = \rho g h_{01}$$

and the pressure in pipe **(2)** is:

$$p_2 = \rho g h_{02}$$

where **(h_{01})** and **(h_{02})** are the heights shown in figure **(3.3(b)).**

We can now write Bernoulli's equation **(3.33)** as:

$$\rho g h_{01} + \frac{1}{2}\rho v_1^2 = \rho g h_{02} + \frac{1}{2}\rho v_2^2$$

Replacing **(v_2)** by its value from the continuity equation **(3.16)**, we get:

$$\rho g h_{01} + \tfrac{1}{2}\rho v_1^2 = \rho g h_{02} + \tfrac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)v_1\right]^2$$

$$\rho g h_{01} - \rho g h_{02} = +\tfrac{1}{2}\rho\frac{A_1^2}{A_2^2}v_1^2 - \tfrac{1}{2}\rho v_1^2$$

$$\rho g\left(h_{01} - h_{02}\right) = +\tfrac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)v_1^2$$

Solving for **(v_1^2)**, we get:

$$v_1^2 = \frac{\rho g(h_\circ 1 - h_\circ 2)}{\frac{1}{2}\rho\left[(A_1^2 / A_2^2) - 1\right]}$$

Solving for **(v_1)** , we get:

$$v_1 = \sqrt{\frac{2g(h_\circ 1 - h_\circ 2)}{(A_1^2 / A_2^2) - 1}} \ldots(3-35)$$

Equation **(3.35)** now gives us a simple means of determining the velocity of fluid flow in a pipe.

The main pipe containing the fluid is opened and the Venturi meter is connected between the opened pipes.

When the fluid starts to move, the heights **(h_{01})** and **(h_{02})** are measured.

Since the cross-sectional areas are easily determined by measuring the diameters of the pipes, the velocity of the fluid flow is easily calculated from equation **(3.35)**.

3.3.2 The Flow of a Liquid Through an Orifice

Let us consider the large tank of water shown in figure **(3.4)** .

Let the top of the fluid be location **(1)** and the orifice be location **(2)**.

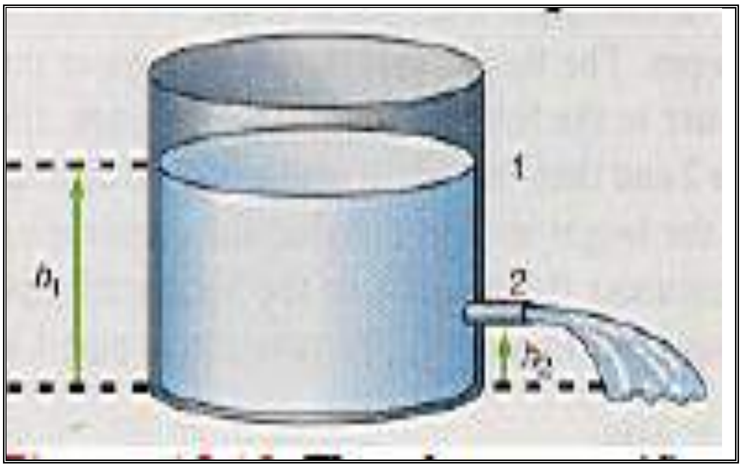


Figure (3.4) : Flow from an orifice.

Bernoulli's theorem applied to the tank, taken from equation **(3.31)**, is :

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^{\,2} = P_o + \rho g h_2 + \frac{1}{2}\rho v_2^{\,2}$$

But the pressure at the top of the tank and the outside pressure at the orifice are both **(p_0)** , the normal atmospheric pressure.

Also, because of the very large volume of fluid, the small loss through the orifice causes an insignificant vertical motion of the top of the fluid.

Thus, $(v_1 \approx 0)$.

Bernoulli's equation becomes :

$$P_o + \rho g h_1 = P_o + \rho g h_2 + \frac{1}{2}\rho {v_2}^2$$

The pressure term (p_0) on both sides of the equation cancels out.

Also (h_2) is very small compared to (h_1) and it can be neglected, leaving;

$$\rho g h_1 = \frac{1}{2}\rho {v_2}^2$$

Solving for the velocity of efflux, we get :

$$\boxed{v_2 = \sqrt{2gh_1} \quad ...(3-36)}$$

Notice that the velocity of efflux is equal to the velocity that an object would acquire when dropped from the height (h_1) .

The Language of Physics

Lecture 3: (Dynamic Fluids)

Law of conservation of mass

In any ordinary mechanical or chemical process, mass is neither created nor destroyed .

The equation of continuity

An equation based on the law of conservation of mass, that indicates that when the cross-sectional area of a pipe gets smaller, the velocity of the fluid must become greater.

Conversely, when the cross-sectional area increases, the velocity of the fluid must decrease .

Bernoulli's theorem

The sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume at any one location of the fluid is equal to the sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume at any other location in the fluid, for a **nonviscous**, **incompressible fluid** in **streamlined flow** .

Venturi effect

The effect of the decrease in pressure with the increase in speed of the fluid in a horizontal pipe .

Venturi meter

A device that uses the Venturi effect to measure the velocity of fluids in pipes .

Summary of Important Equations

Lecture 3: (Dynamic Fluids)

Mass flow rate	$\frac{\Delta m}{\Delta t} = \rho A v$
Equation of continuity	$A_1 v_1 = A_2 v_2$
Work done in moving a fluid	$W = p\Delta V$
Bernoulli's theorem	$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$
Velocity of flow in Venturi meter	$v_1 = \sqrt{\frac{2g(h_\circ 1 - h_\circ 2)}{(A_1^2 / A_2^2) - 1}}$
The velocity of efflux through an Orifice	$v_2 = \sqrt{2gh_1}$

Problems for Lecture 3

(Dynamic Fluids)

Problem 3.1

Flow rate

What is the mass flow rate of water in a pipe whose diameter ***d*** is **(10 cm)** when the water is moving at a velocity of **(0.322 m/s)**.

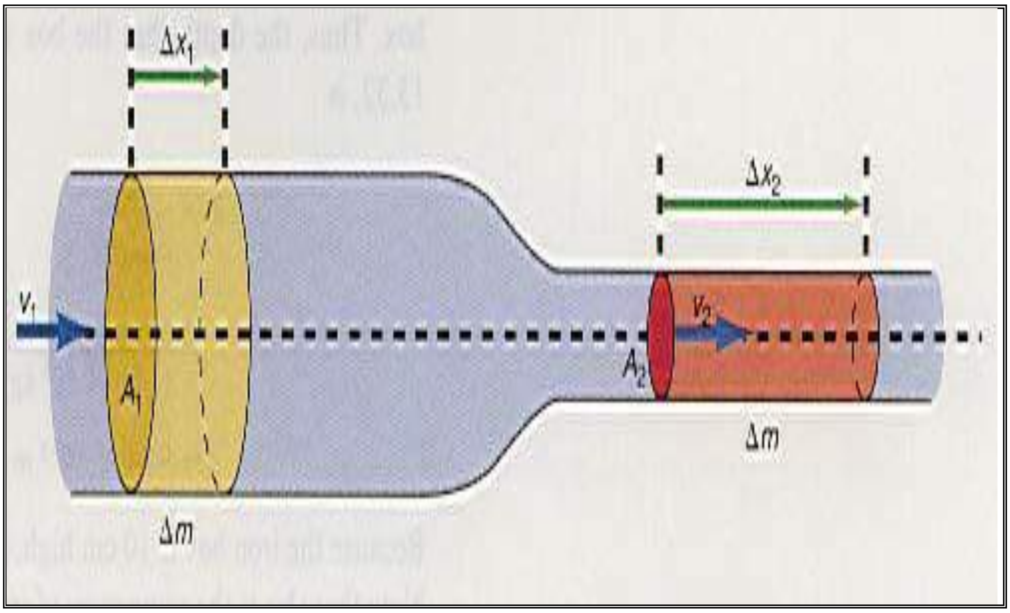
Answer : (2.53 kg/s)

Problem 3.2

Applying the equation of continuity

In problem **3.1** the cross-sectional area A_1 was **(7.85×10^{-3} m²)** and the velocity v_1 was **(0.322 m/s)**. If the diameter of the pipe to the right in figure below is **(4 cm)**, find the velocity of the fluid in the right-hand pipe.

Answer : (2.01 m/s)

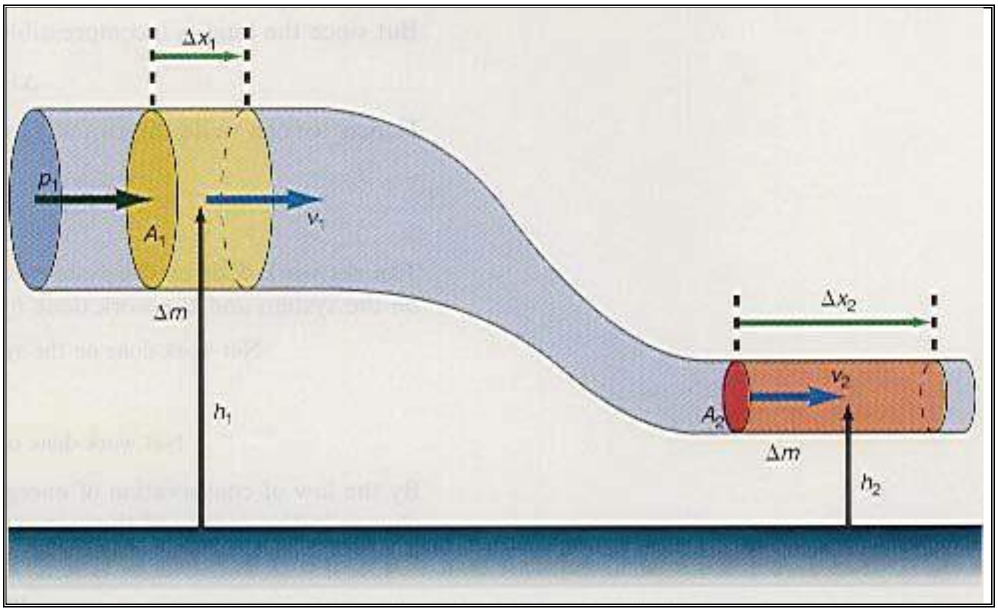


Problem 3.3

Applying Bernoulli's theorem

In figure below, the pressure **($p_1 = 2.94 \times 10^3$ N/m²)**, whereas the velocity of the water is **($v_1 = 0.322$ m/s)**. The diameter of the pipe at location **1** is **(10 cm)** and it is **(5 m)** above the ground. If the diameter of the pipe at location **2** is **(4 cm)**, and the pipe is **(2 m)** above the ground, find the velocity of the water **v_2** at position **2**, and the pressure **p_2** of the water at position **2**.

Answer : (2.01 m/s , 3.04 x 10^4 N/m²)



Problem 3.4

When the velocity increases, the pressure decreases

In problem **3.2**, the velocity v_1 in area A_1 was **(0.322 m/s)** and the velocity v_2 in area A_2 was found to be **(2.01 m/s).** If the pressure in the left pipe is **(2.94 x 10^3 Pa)**, what is the pressure p_2 in the constricted pipe?

Answer : (9.7 x 10^2 N/m^2)

Problem 3.5

A Venturi meter

A Venturi meter reads heights of **(h_{01} = 30 cm)** and **(h_{02} = 10 cm).** Find the velocity of flow v_1 in the pipe. The area **(A_1 = 7.85 x 10^{-3} m^2)** and the area of **(A_2 = 1.26 x 10^{-3} m^2).**

Answer : (0.322 m/s)

Problem 3.6

The velocity of efflux

A large water tank, **(10 m)** high, springs a leak at the bottom of the tank. Find the velocity of the escaping water.

Answer : (14 m/s)