Kirkuk University

Science College

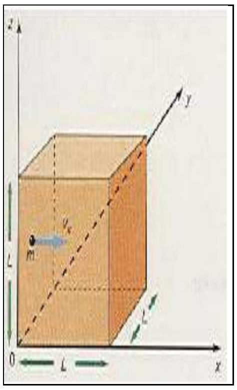
Physics Department

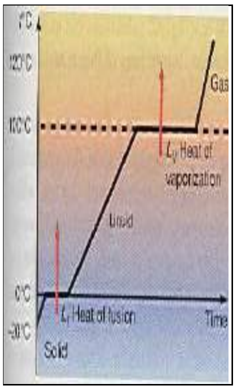
Lectures of

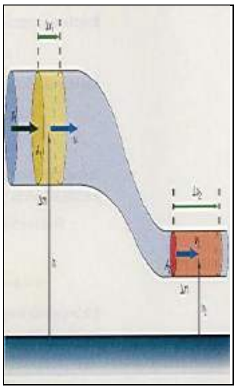
Properties of Matter

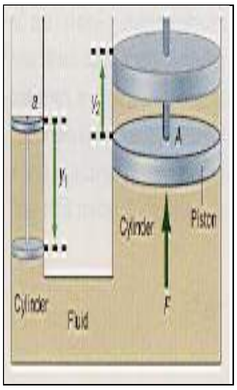
Lecture =1=

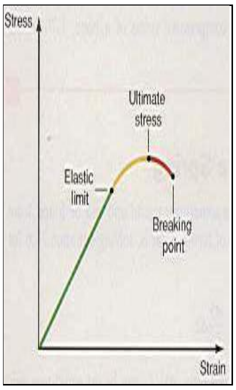
Elasticity











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Lecture 1: (Elasticity)

1.1 The Atomic Nature of Elasticity

Elasticity *is that property of a body by which it experiences a change in size or shape whenever a deforming force acts on the body.*

When the force is removed the body returns to its original size and shape.

Most people are familiar with the stretching of a rubber band.

All materials, however, have this same elastic property, but in most materials it is not so pronounced.

The explanation of the elastic property of solids is found in an atomic description of a solid.

Most solids are composed of a very large number of atoms or molecules arranged in a fixed pattern called the **lattice structure of a solid** and shown schematically in figure **(1.1)**.

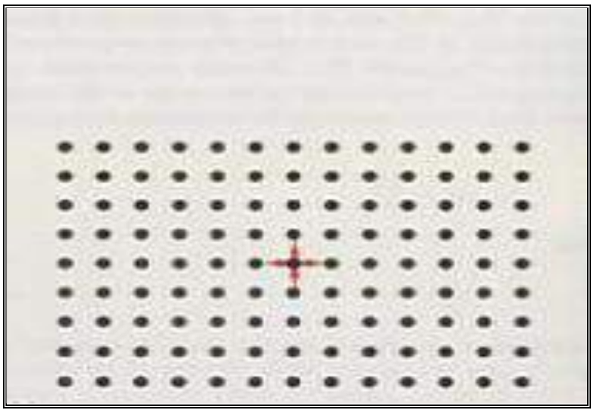


Figure (1.1) **: Lattice structure of a solid.**

These atoms or molecules are held in their positions by electrical forces.

The electrical force between the molecules is attractive and tends to pull the molecules together.

Thus, the solid resists being pulled apart.

Any one molecule in figure **(1.1)** has an attractive force pulling it to the right and an equal attractive force pulling it to the left.

There are also equal attractive forces pulling the molecule up and down, and in and out.

A repulsive force between the molecules also tends to repel the molecules if they get too close together.

This is why solids are difficult to compress.

The net result of all these molecular forces is that each molecule is in a position of equilibrium.

If we try to pull one side of a solid material to the right, let us say, then we are in effect pulling all these molecules slightly away from their equilibrium position.

The displacement of any one molecule from its equilibrium position is quite small, but since there are billions of molecules, the total molecular displacements are directly measurable as a change in length of the material.

When the applied force is removed, the attractive molecular forces pull all the molecules back to their original positions, and the material returns to its original length.

If we now exert a force on the material in order to compress it, we cause the molecules to be again displaced from their equilibrium position, but this time they are pushed closer together.

The repulsive molecular force prevents them from getting too close together, but the total molecular displacement is directly measurable as a reduction in size of the original material.

When the compressive force is removed, the repulsive molecular force causes the atoms to return to their equilibrium position and the solid returns to its original size.

Hence, the elastic properties of matter are a manifestation of the molecular forces that hold solids together.

1.2 Hooke's Law — Stress and Strain

If we apply a force to a rubber band, we find that the rubber band stretches.

Similarly, if we attach a wire to a support, as shown in figure **(1.2)**, and sequentially apply forces of magnitude ***F*, 2*F*,** and **3*F*** to the wire, we find that the wire stretches by an amount **Δ*L*, 2Δ*L*,** and **3Δ*L*,** respectively.

(Note that the amount of stretching is greatly exaggerated in the diagram for illustrative purposes.)

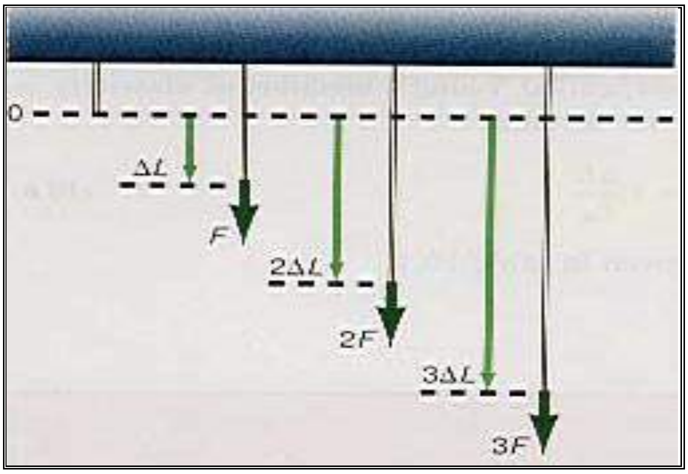


Figure (1.2) **: Stretching an object.**

The deformation, **(Δ*L*)**, is directly proportional to the magnitude of the applied force ***(F)*** and is written mathematically as:

$$\boxed{\Delta L \propto F \quad ...(1-1)}$$

This aspect of elasticity is true for all solids.

It would be tempting to use equation **(1.1)** as it stands to formulate a theory of elasticity, but with a little thought it becomes obvious that although it is correct in its description, it is incomplete.

Let us consider two wires, one of cross-sectional area ***A,*** and another with twice that area, namely **2*A*,** as shown in figure **(1.3)**.

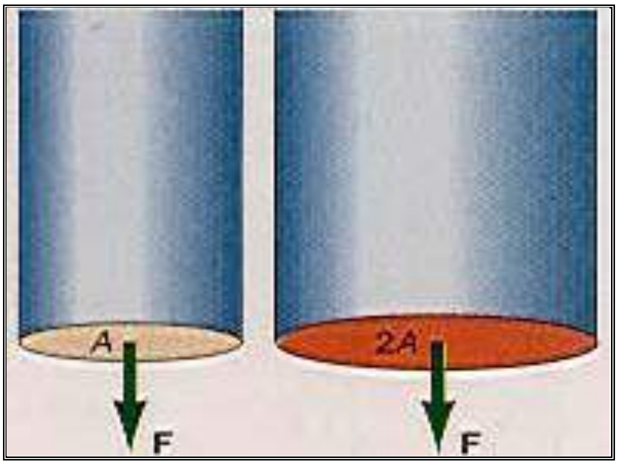


Figure (1.3): **The deformation is inversely proportional to the cross-sectional area of the wire**

When we apply a force **F** to the first wire, that force is distributed over all the atoms in that cross-sectional area ***A***.

If we subject the second wire to the same applied force **F,** then this same force is distributed over twice as many atoms in the area **2*A*** as it was in the area ***A***.

Equivalently we can say that each atom receives only half the force in the area **2*A*** that it received in the area ***A***.

Hence, the total stretching of the **2*A*** wire is only **1/2** of what it was in wire ***A***.

Thus, the elongation of the wire ***(ΔL)*** is inversely proportional to the cross-sectional area ***(A)*** of the wire, and this is written:

$$\Delta L \propto \frac{1}{A} ...(1-2)$$

Note also that the original length of the wire must have something to do with the amount of stretch of the wire.

For if a force of magnitude ***F*** is applied to two wires of the same cross-sectional area, but one has length L_0 and the other has length $2L_0$, the same force is transmitted to every molecule in the length of the wire.

But because there are twice as many molecules to stretch apart in the wire having length $2L_0$, there is twice the deformation, or $2\Delta L$, as shown in figure **(1.4)**.

We write this as the proportion:

$$\Delta L \propto L_o ...(1-3)$$

The results of equations **(1.1)**, **(1.2)** and **(1.3)** are, of course, also deduced experimentally.

The deformation **(ΔL)** *of the wire is thus* ***directly*** *proportional to the magnitude of the applied force* ***(F)*** *(equation 1.1),****inversely*** *proportional to the cross-sectional area* ***(A)*** *(equation 1.2), and* ***directly*** *proportional to the original length of the wire* ***(L₀)*** *(equation 1.3).*

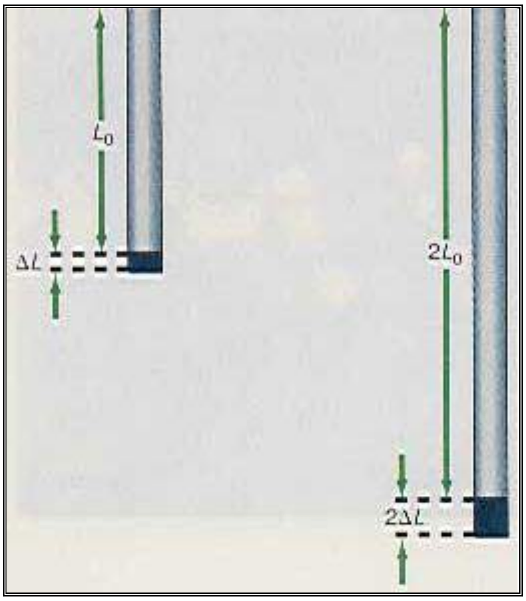


Figure (1.4) **: The deformation is directly proportional to the original length of the wire.**

These results can be incorporated into the one proportionality**:**

$$\Delta L \propto \frac{FL_o}{A}$$

which we rewrite in the form:

$$\boxed{\frac{F}{A} \propto \frac{\Delta L}{L_o} ...(1-4)}$$

The ratio of the magnitude of the applied force to the cross-sectional area of the wire is called the ***stress*** *acting on the wire, while the ratio of the change in length to the original length of the wire is called the* ***strain*** *of the wire.*

Equation **(1.4)** is a statement of ***Hooke's law of elasticity,*** *which says that in an elastic body the stress is directly proportional to the strain,* that is:

$$\boxed{Stress \propto Strain ...(1-5)}$$

The stress is what is applied to the body, while the resulting effect is called the strain.

To make an equality out of this proportion, we must introduce a constant of proportionality.

This constant depends on the type of material used, since the molecules, and hence the molecular forces of each material, are different.

This constant, called **Young's modulus of elasticity** is denoted by the letter ***(Y)*** .

Equation **(1.4)** thus becomes:

$$\boxed{\frac{F}{A} = Y.\frac{\Delta L}{L_o} ...(1-6)}$$

The value of ***(Y)*** for various materials is given in table **(1.1)**.

Table : 1-1

Some Elastic Constants

Substance	Young's Modulus	Shear Modulus	Bulk Modulus	Elastic Limit	Ultimate Tensile Stress
	$N/m^2 \times 10^{10}$	$N/m^2 \times 10^{10}$	$N/m^2 \times 10^{10}$	$N/m^2 \times 10^8$	$N/m^2 \times 10^8$
Aluminum	7.0	3	7	1.4	1.4
Bone	1.5	8.0			1.30
Brass	9.1	3.6	6	3.5	4.5
Copper	11.0	4.2	14	1.6	4.1
Iron	9.1	7.0	10	1.7	3.2
Lead	1.6	0.56	0.77		0.2
Steel	21	8.4	16	2.4	4.8

The applied stress on the wire cannot be increased indefinitely if the wire is to remain elastic.

Eventually a point is reached where the stress becomes so great that the atoms are pulled permanently away from their equilibrium position in the lattice structure.

This point is called the **elastic limit** of the material and is shown in figure **(1.5)**.

When the stress exceeds the elastic limit the material does not return to its original size or shape when the stress is removed.

The entire lattice structure of the material has been altered.

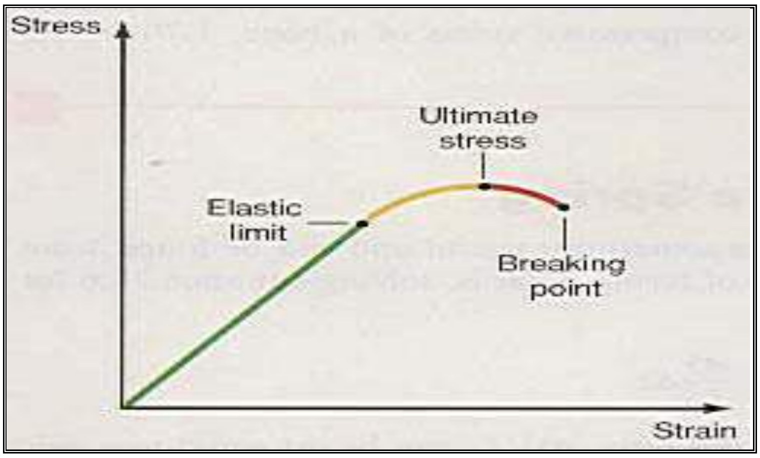


Figure (1.5) **: Stress-strain relationship.**

If the stress is increased beyond the elastic limit, eventually the ***ultimate stress*** point is reached.

This is the highest point on the stress-strain curve and represents the greatest stress that the material can bear.

Brittle materials break suddenly at this point, while some ductile materials can be stretched a little more due to a decrease in the cross-sectional area of the material.

But they too break shortly thereafter at the ***breaking point***.

Hooke's law is only valid below the elastic limit, and it is only that region that will concern us.

Although we have been discussing the stretching of an elastic body, a body is also elastic under compression.

If a large load is placed on a column, then the column is compressed, that is, it shrinks by an amount **(ΔL)**.

When the load is removed the column returns to its original length.

1.3 Hooke's Law for a Spring

A simpler formulation of Hooke's law is sometimes useful and can be found from equation **(1.6)** by a slight rearrangement of terms.

That is, solving equation **(1.6)** for ***(F)*** gives:

$$F = \frac{AY}{L_o}\Delta L$$

Because ***(A)**, **(Y)**,* and **(L_0)** are all constants, the term **(AY/L_0)** can be set equal to a new constant ***(k)**,* namely:

$$\boxed{k = \frac{AY}{L_o} \quad ...(1-7)}$$

We call ***(k)*** a force constant or a spring constant.

Then:

$$\boxed{F = k\Delta L \quad ...(1-8)}$$

The change in length **(ΔL)** of the material is simply the final length **(L)** minus the original length **(L_0)**.

We can introduce a new reference system to measure the elongation, by calling the location of the end of the material in its un stretched position, **($x = 0$)**.

Then we measure the stretch by the value of the displacement ***(x)*** from the un stretched position, as seen in figure **(1.6)**.

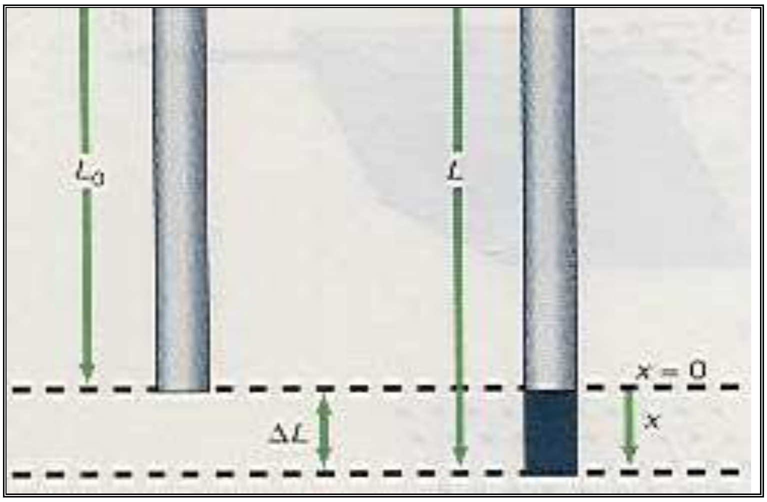


Figure (1.6) **: Changing the reference system.**

Thus, **(ΔL)** = ***x***, in the new reference system, and we can write equation **(1.8)** as:

$$\boxed{F = kx \ldots (1-9)}$$

Equation **(1.9)** is a simplified form of Hooke's law that we use in vibratory motion containing springs.

For a helical spring, we cannot obtain the spring constant from equation **(1.7)** because the geometry of a spring is not the same as a simple straight wire.

However, we can find ***(k)*** experimentally by adding various weights to a spring and measuring the associated elongation ***x***, as seen in figure **1.7(a).**

A plot of the magnitude of the applied force ***(F)*** versus the elongation ***(x)*** gives a straight line that goes through the origin, as in figure **(1.7(b)).**

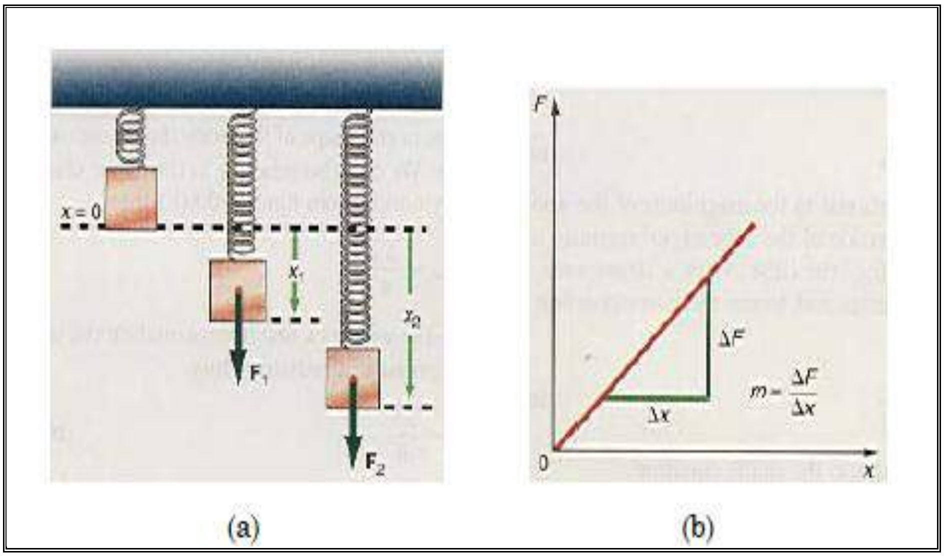


Figure (1.7): **Experimental determination of a spring constant.**

Because Hooke's law for the spring, equation **(1.9)**, is an equation of the form of a straight line passing through the origin, that is:

$$y = mx$$

the slope ***(m)*** of the straight line is the spring constant ***(k).***

In this way, we can determine experimentally the spring constant for any spring.

1.4 Elasticity of Shape - Shear

In addition to being stretched or compressed, a body can be deformed by changing the shape of the body.

If the body returns to its original shape when the distorting stress is removed, the body exhibits the property of elasticity of shape, sometimes called **shear**.

As an example, consider the cube fixed to the surface in figure **(1.8(a)).**

A tangential force **(F_t)** is applied at the top of the cube, a distance ***(h)*** above the bottom.

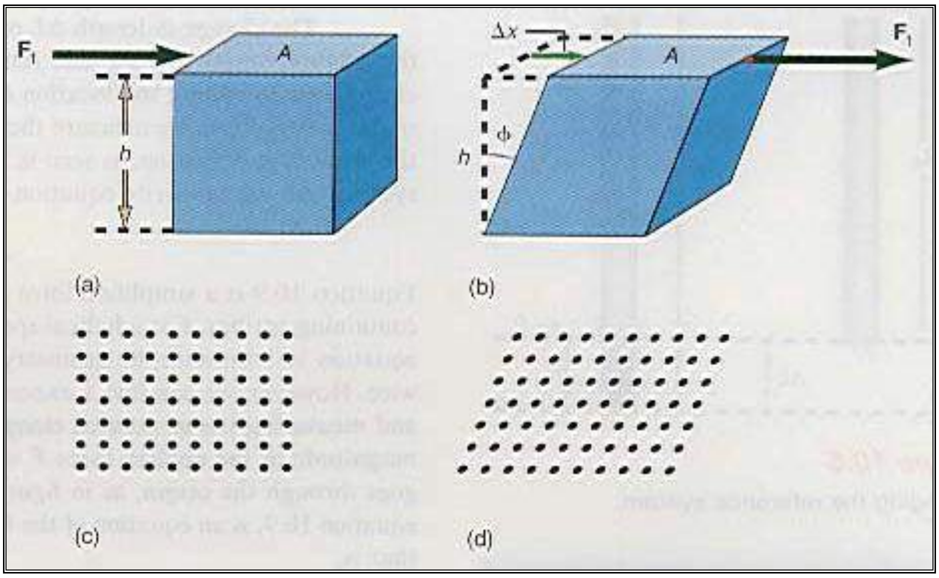


Figure (1.8) **: Elasticity of shear.**

The magnitude of this force ***(F_t)*** times the height ***(h)*** of the cube would normally cause a torque to act on the cube to rotate it.

However, since the cube is not free to rotate, the body instead becomes deformed and changes its shape, as shown in figure **(1.8(b)).**

The normal lattice structure is shown in figure **(1.8(c)),** and the deformed lattice structure in figure **(1.8(d)).**

The tangential force applied to the body causes the layers of atoms to be displaced sideways; one layer of the lattice structure slides over another.

The tangential force thus causes a change in the shape of the body that is measured by the angle ϕ, called the *angle of shear*.

We can also relate ϕ to the linear change from the original position of the body by noting from **(**figure **1.8(b))** that:

$$\tan\phi = \frac{\Delta x}{h}$$

Because the deformations are usually quite small, as a first approximation the **tan** ϕ can be replaced by the angle ϕ itself, expressed in radians.

Thus:

$$\boxed{\phi = \frac{\Delta x}{h} \ldots (1-10)}$$

*Equation **(1.10)** represents the **shearing strain** of the body.*

The tangential force **(F_t)** causes a deformation ϕ of the body and we find experimentally that:

$$\boxed{\phi \propto F_t \ldots (1-11)}$$

That is, the angle of shear is directly proportional to the magnitude of the applied tangential force **(F_t)**.

We also find the deformation of the cube experimentally to be inversely proportional to the area of the top of the cube.

With a larger area, the distorting force is spread over more molecules and hence the corresponding deformation is less.

Thus:

$$\phi \propto \frac{1}{A} \ldots(1-12)$$

Equations **(1.11)** and **(1.12)** can be combined into the single equation:

$$\phi \propto \frac{F_t}{A} \ldots(1-13)$$

Note that ***(Ft/A)*** has the dimensions of a stress and it is now defined as the **shearing stress:**

$$ShearingStress = \frac{F_t}{A} \ldots(1-14)$$

Since (ϕ) is the shearing strain, equation **(1.13)** shows the familiar proportionality that stress is directly proportional to the strain.

Introducing a constant of proportionality (***S***)*,* called the **shear modulus,** Hooke's law for the elasticity of shear is given by:

$$\frac{F_t}{A} = S\phi \ldots(1-15)$$

Values of ***(S)*** for various materials are given in table **(1.1)**.

The larger the value of ***(S)*** *,* the greater the resistance to shear.

Note that the shear modulus is smaller than Young's modulus ***(Y).***

This implies that it is easier to slide layers of molecules over each other than it is to compress or stretch them.

The shear modulus is also known as the *torsion modulus* and the *modulus of rigidity.*

1.5 Elasticity of Volume

If a uniform force is exerted on all sides of an object, as in figure **(1.9)**, such as a block under water, each side of the block is compressed.

Thus, the entire volume of the block decreases.

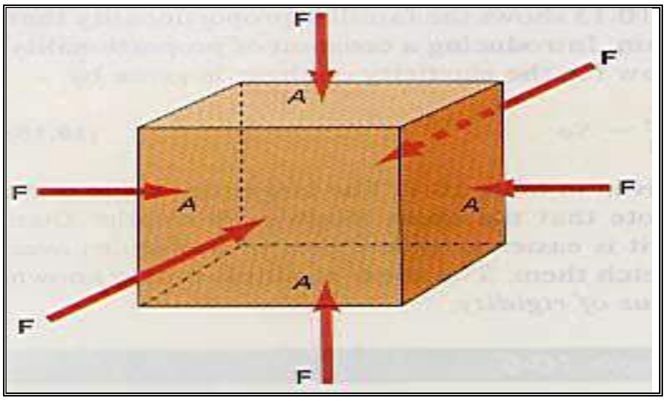


Figure (1.9) : **Volume elasticity.**

The **compressional stress** is defined as:

$$Stress = \frac{F}{A} \quad ...(1-16)$$

where ***(F)*** is the magnitude of the normal force acting on the cross-sectional area ***(A)*** of the block.

The **strain** is measured by the change in volume per unit volume, that is:

$$Strain = \frac{\Delta V}{V_o} \quad ...(1-17)$$

Since the stress is directly proportional to the strain, by Hooke's law, we have:

$$\frac{F}{A} \propto \frac{\Delta V}{V_o} \quad ...(1-18)$$

To obtain an equality, we introduce a constant of proportionality ***(B),*** called the **bulk modulus,** and Hooke's law for **elasticity of volume** becomes:

$$\frac{F}{A} = -B\frac{\Delta V}{V_o} \quad ...(1-19)$$

The minus sign is introduced in equation **(1.19)** because an increase in the stress **(*F*/*A*)** causes a decrease in the volume, leaving **(Δ*V*)** negative.

The bulk modulus is a measure of how difficult it is to compress a substance.

The reciprocal of the bulk modulus ***(B)*** , called the **compressibility** ***(k)*** , is a measure of how easy it is to compressthe substance.

The bulk modulus ***(B)*** is used for solids, while the compressibility ***(k)*** is usually used for liquids.

Quite often the body to be compressed is immersed in a liquid.

In dealing with liquids and gases it is convenient to deal with the pressure exerted by the liquid or gas.

We will see in detail in Lecture **2** that pressure is defined as the force that is acting over a unit area of the body, that is:

$$P = \frac{F}{A}.$$

For the case of volume elasticity, the stress ***(F/A)****,* acting on the body by the fluid, can be replaced by the pressure of the fluid itself.

Thus, Hooke's law for volume elasticity can also be written as:

$$\boxed{P = -\frac{B\Delta V}{V_o} \ldots(1-20)}$$

The Language of Physics

Lecture 1: (Elasticity)

Elasticity

That property of a body by which it experiences a change in size or shape whenever a deforming force acts on the body.

The elastic properties of matter are a manifestation of the molecular forces that hold solids together .

Lattice structure of a solid

A regular, periodically repeated, three-dimensional array of the atoms or molecules comprising the solid .

Stress

For a body that can be either stretched or compressed, the stress is the ratio of the applied force acting on a body to the cross-sectional area of the body .

Strain

For a body that can be either stretched or compressed, the ratio of the change in length to the original length of the body is called the strain .

Hooke's law

In an elastic body, the stress is directly proportional to the strain .

Young's modulus of elasticity

The proportionality constant in Hooke's law.

It is equal to the ratio of the stress to the strain .

Elastic limit

The point where the stress on a body becomes so great that the atoms of the body are pulled permanently away from their equilibrium position in the lattice structure.

When the stress exceeds the elastic limit, the material will not return to its original size or shape when the stress is removed.

Hooke's law is no longer valid above the elastic limit .

Shear

That elastic property of a body that causes the shape of the body to be changed when a stress is applied.

When the stress is removed the body returns to its original shape .

Shearing strain

The angle of shear, which is a measure of how much the body's shape has been deformed .

Shearing stress

The ratio of the tangential force acting on the body to the area of the body over which the tangential force acts .

Shear modulus

The constant of proportionality in Hooke's law for shear.

It is equal to the ratio of the shearing stress to the shearing strain .

Bulk modulus

The constant of proportionality in Hooke's law for volume elasticity.

It is equal to the ratio of the compressional stress to the strain.

The strain for this case is equal to the change in volume per unit volume .

Elasticity of volume

When a uniform force is exerted on all sides of an object, each side of the object becomes compressed.

Hence, the entire volume of the body decreases.

When the force is removed the body returns to its original volume .

Compressibility

The reciprocal of the bulk modulus ***(B)***.

Summary of Important Equations

Lecture 1: (Elasticity)

Hooke's law in general	stress ∝ strain
Hooke's law for stretching or compression	$\frac{F}{A} = Y \frac{\Delta L}{L_0}$
Hooke's law for a spring	$F = kx$
Shearing strain	$\phi = \frac{\Delta x}{h}$
Hooke's law for shear	$\frac{F_t}{A} = S\phi$
Hooke's law for volume elasticity	$\frac{F}{A} = -B \frac{\Delta V}{V_0}$ $p = -B\frac{\Delta V}{V_0}$

Problems for Lecture 1

(Elasticity)

Problem 1.1

Stretching a wire

A steel wire **(1 m)** long with a diameter **(d = 1 mm)** has a **(10 kg)** mass hung from it. The value of **Y** for steel is **(21×10^{10} N/m^2)**.

(a) How much will the wire stretch**?** **Answer : (0.594×10^{-3} m)**

(b) What is the stress on the wire**?** **Answer : (1.25×10^8 N/m^2)**

(c) What is the strain**?** **Answer : (0.594×10^{-3})**

Problem 1.2

Compressing a steel column

A **(445000) N** load is placed on top of a steel column **(3.05 m)** long and **(10.2 cm)** in diameter. By how much is the column compressed**?** The value of **Y** for steel is **(21×10^{10} N/m^2).**

Answer : (7.91×10^{-4} m)

Problem 1.3

Exceeding the ultimate compressive strength

A human bone is subjected to a compressive force of **(5×10^5 N)**. The bone has an approximate area of **(4 cm^2).** If the ultimate compressive strength for a bone is **(1.70×10^8 N/m^2)**, will the bone be compressed or will it break under this force**?**

Answer : (12.5×10^8 N/m^2)

Problem 1.4

The elongation of a spring

A spring with a force constant of **(50 N/m)** is loaded with a **(0.500 kg)** mass. Find the elongation of the spring. **Answer : (0.098 m)**

Problem 1.5

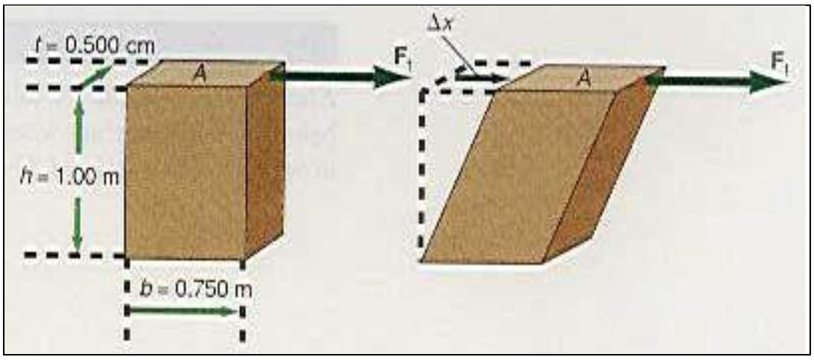
Elasticity of shear

A sheet of copper **(0.750 m)** long, **(1 m)** high, and **(0.500 cm)** thick is acted on by a tangential force of **(50000 N)**, as shown in figure below. The value of **S** for copper is **(4.20×10^{10} N/m^2).** Find**:**

(a) The shearing stress **? Answer : (1.33×10^7 N/m^2)**

(b) The shearing strain**? Answer : (3.17×10^{-4})**

(c) The linear displacement **Δx? Answer : (3.17×10^{-4} m)**



Problem 1.6

Elasticity of volume

A solid copper sphere of **(0.500 m^3)** volume is placed **(30.5 m)** below the ocean surface where the pressure is **(3.00×10^5 N/m^2)**. What is the change in volume of the sphere**?** The bulk modulus for copper is **(14×10^{10} N/m^2)**.

Answer : (-1.1×10^{-6} m^3)