Kirkuk University

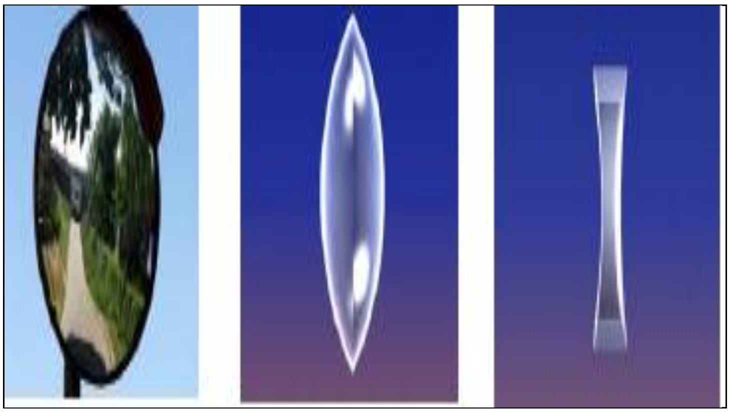
Science College

Physics Department

Lectures of

GEOMETRIC OPTICS

Lecture – 10 –



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Lecture 10: Fermat's Principle

10 – 1 Fermat's principle

- Fermat's principle states that " **light travels between two points along the path that requires the least time, as compared to other nearby paths**."

- From Fermat's principle, one can derive (**a**) the law of reflection [**the angle of incidence is equal to the angle of reflection**] and (**b**) the law of refraction [**Snell's law**]. The derivations are given below.

10 - 2 Fermat's Principle and the Reflection Law

- Figure **(10-1)** shows two fixed points **A** and **B** and a reflecting ray **APB** connecting them.

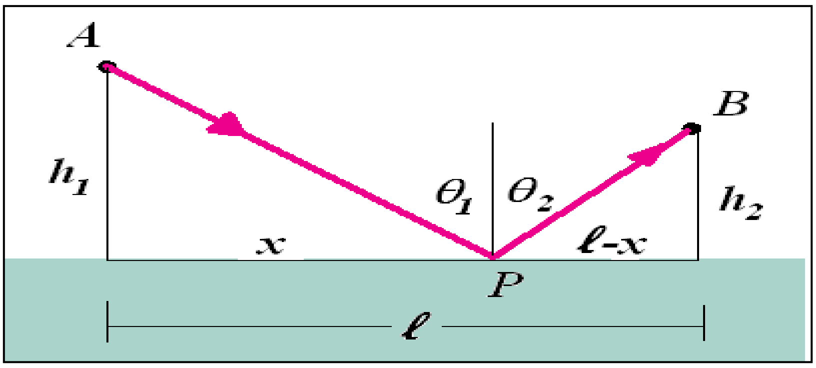

Fig.(10-1) : The reflection of a plane wave at a plane interface as analyzed using Fermat's principle . A ray from **A** passes through **B** after reflection at **P** .

- The total length **L** of this ray is **:**

$$L = \sqrt{x^2 + h_1^{\ 2}} + \sqrt{(\ell - x)^2 + h_2^{\ 2}} \;...(1)$$

- According to Fermat's principal , **P** will have a position such that the time of travel $t = \frac{L}{c}$ of the light must be a least time , which occurs when $dt/dx = 0$.

- Taking this derivative yields **:**

$$\frac{dt}{dx} = \frac{1}{c}.\frac{dL}{dx} \;...(2)$$

$$\Rightarrow \frac{dt}{dx} = \frac{1}{2c}(x^2 + h_1^{\ 2})^{-1/2}.(2x) + \frac{1}{2c}\left[(\ell - x)^2 + h_2^{\ 2}\right]^{-1/2}.(2).(\ell - x).(-1) = 0$$

which we can write as **:**

$$\frac{x}{\sqrt{x^2 + h_1^{\ 2}}} = \frac{(\ell - x)}{\sqrt{(\ell - x)^2 + h_2^{\ 2}}} \;...(3)$$

- Comparison with Fig.(**10-1**) shows that we can write this as **:**

$$\sin\theta_1 = \sin\theta_2$$

$$\boxed{\theta_1 = \theta_2}$$

which is the law of reflection**.**

10 - 3 Fermat's Principle and the Refraction Law

- To prove the law of refraction from Fermat's principle, consider Fig.(**10-2**), which shows two fixed points **A** and **B** in two different media and a refracting ray **APB** connecting them **.**

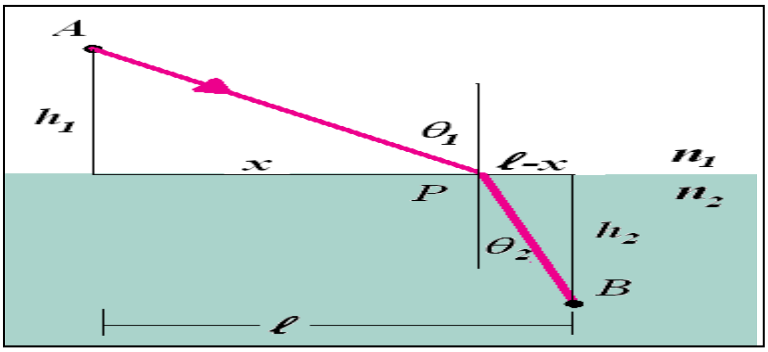

Fig.(10-2) : The refraction of a plane wave at a plane interface as analyzed using Fermat's principle . A ray from **A** passes through **B** after refraction at **P** .

- The time **t** for the ray to travel from **A** to **B** is given by the Eq.(**4**) **:**

$$t = \frac{L_1}{v_1} + \frac{L_2}{v_2} \ldots(4)$$

- Using the relation $n = \frac{c}{v}$ we can write this as the Eq.(**5**) **:**

$$t = \frac{n_1.L_1 + n_2.L_2}{c} = \frac{L}{c} \ldots(5)$$

- Where **L** is the optical path length , defined as Eq.(**6**) **:**

$$L = n_1.L_1 + n_2.L_2 \ldots(6)$$

- For any light ray traveling through successive media, the optical path length is the sum of the products of the geometrical path length of each segment and the index of refraction of that medium**.**

- Fermat's principal requires that the time **t** for the light to travel the path **APB** must be a least time, which in turn requires that **x** be chosen so that $dt/dx = 0$**.**

- The optical path length in Fig.(**10-2**) is **:**

$$L = n_1.L_1 + n_2.L_2 = n_1.\sqrt{x^2 + h_1^{\ 2}} + n_2.\sqrt{(\ell - x)^2 + h_2^{\ 2}} \quad ...(7)$$

- Substituting this result into Eq.(**2**) and differentiating we obtain **:**

$$\frac{dt}{dx} = \frac{1}{c}.\frac{dL}{dx} \quad ...(8)$$

$$\Rightarrow \frac{dt}{dx} = \frac{n_1}{2c}(x^2 + h_1^{\ 2})^{-1/2}.(2x) + \frac{n_2}{2c}\left[(\ell - x)^2 + h_2^{\ 2}\right]^{-1/2}.(2).(\ell - x).(-1) = 0$$

which we can write as **:**

$$n_1.\frac{x}{\sqrt{x^2 + h_1^{\ 2}}} = n_2.\frac{(\ell - x)}{\sqrt{(\ell - x)^2 + h_2^{\ 2}}} \quad ...(9)$$

- Comparison with Fig.(**10-2**) shows that we can write this as **:**

$$\boxed{n_1.\sin\theta_1 = n_2.\sin\theta_2}$$

which is the law of refraction (**Snell's law**) **.**