Kirkuk University

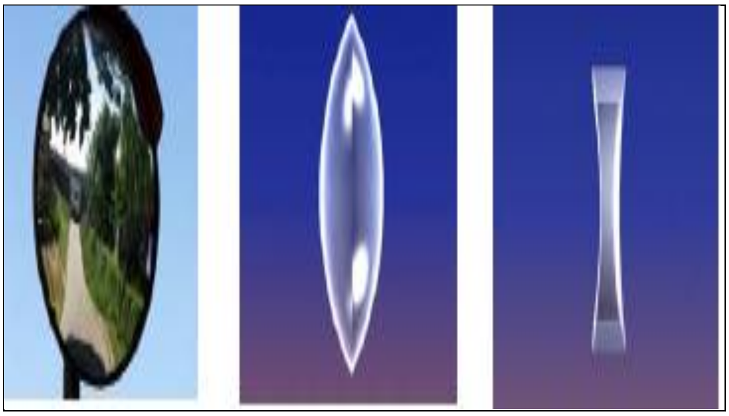
Science College

Physics Department

Lectures of

GEOMETRIC OPTICS

Lecture – 18 –



Assistant professor Dr.Jawdet Hedayet Mohammed

Lecturer in Kirkuk University

Science College – Physics Department

Lecture 18: Lenses Formula and Lens maker's Equation

- Is a relationship between the focal length (**f**) , the index of refraction (**n**) of the lens , and the radii of curvature ($\mathbf{r_1}$) and ($\mathbf{r_2}$) of the lens surfaces **.**
- We use the principle that an image formed by one reflecting or refracting surface can serve as the object for a second reflecting or refracting surface**.**
- Considering the ray diagram of refraction for **2** spherical surfaces as shown in Fig.(**18-1**) below**.**

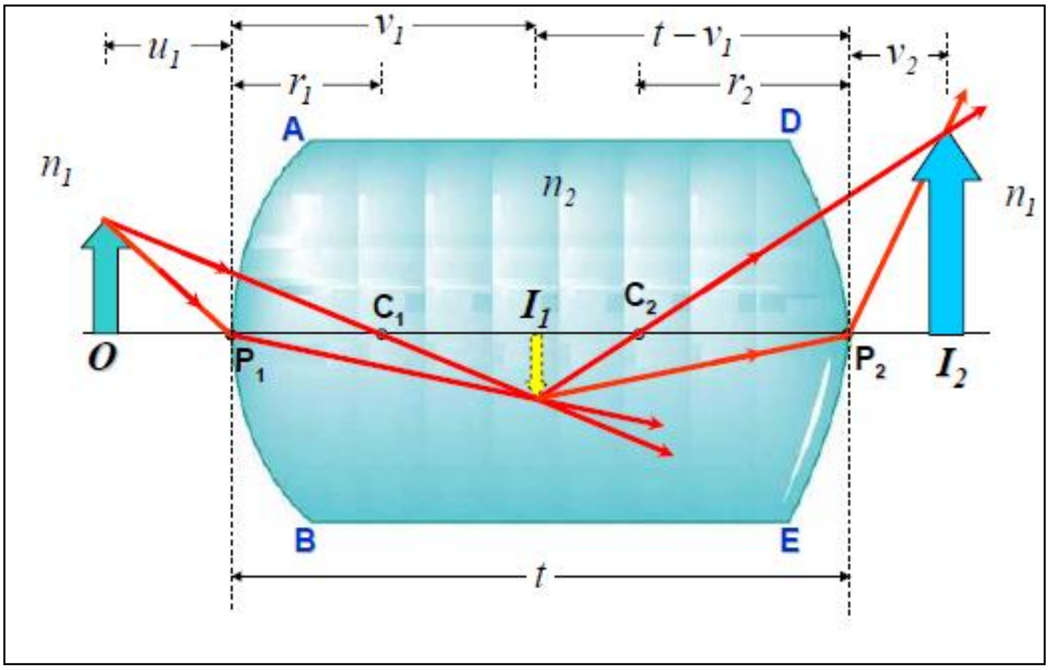


Fig.(18-1) : Ray diagram of refraction for 2 spherical surfaces

- By using the equation of spherical refracting surface, the refraction by first surface **AB** and second surface **DE** are given by **:**

- Convex surface **AB** (**$r = +r_1$**) **:**

$$\frac{n_1}{u_1}+\frac{n_2}{v_1}=\frac{(n_2-n_1)}{r_1}...(1)$$

- Concave surface **DE** (**$r = -r_2$**) **:**

$$\frac{n_2}{(t-v_1)}+\frac{n_1}{v_2}=\frac{(n_1-n_2)}{-r_2}$$

- Assuming the lens is very thin thus ***t* = *0*** **:**

$$\frac{n_2}{-v_1}+\frac{n_1}{v_2}=\frac{(n_1-n_2)}{-r_2}$$

$$\frac{n_2}{v_1}=-\left[(\frac{n_1-n_2}{-r_2})-\frac{n_1}{v_2}\right]$$

$$\frac{n_2}{v_1}=\frac{n_1}{v_2}-(\frac{n_2-n_1}{r_2})...(2)$$

- By substituting eq. (**2**) into eq. (**1**), thus**:**

$$\frac{n_1}{u_1}+\left[\frac{n_1}{v_2}-(\frac{n_2-n_1}{r_2})\right]=\frac{(n_2-n_1)}{r_1}$$

$$\frac{n_1}{u_1}+\frac{n_1}{v_2}=\frac{(n_2-n_1)}{r_1}+\frac{(n_2-n_1)}{r_2}$$

- Then **:**

$$\frac{1}{u_1}+\frac{1}{v_2}=(\frac{n_2}{n_1}-1)(\frac{1}{r_1}+\frac{1}{r_2})...(3)$$

➢ If $u_1 = \infty$ and $v_2 = f$ hence eq. (**3**) becomes **:**

$$\frac{1}{f} = (\frac{n_2}{n_1} - 1)(\frac{1}{r_1} + \frac{1}{r_2}) ...(4)$$

➢ Then equation (**4**) represented **equation of Len's maker** , where **:**

f **:** focal length **.**

r_1**:** radius of curvature of first refracting surface **.**

n_1**:** refractive index of the medium **.**

n_2**:** refractive index of the lens material **.**

r_2**:** radius of curvature of second refracting surface **.**

➢ By equating eq. (**3**) with lens maker's equation, hence **:**

$$\frac{1}{u_1} + \frac{1}{v_2} = \frac{1}{f}$$

➢ Therefore in general**:**

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} ...(5)$$

➢ Then equation (**5**) represented **equation of Lens.**

- Note , if the medium is **air** ($n_1 = n_{air} = 1$) thus the **lens maker's equation** will be**:**

$$\frac{1}{f} = (n-1)(\frac{1}{r_1} + \frac{1}{r_2}) \ldots (6)$$

Where **:**

n : refractive index of the lens material **.**

- An expression for the focal length (**f**) of a lens in terms of its index of refraction (**n**) and the radii of curvature ($\mathbf{r_1}$) and ($\mathbf{r_2}$) of its surfaces **.**

Example:

A biconvex lens is made of glass with refractive index (1.52) having the radii of curvature of $(20cm)$ respectively. Calculate the focal length of the lens in:

a. Water.

b. Carbon disulfide.

(Given $(n_w = 1.33)$ and $(n_c = 1.63)$)

Solution:

$$r_1=+20\ cm,\ r_2=+20\ cm,\ n_g=n_2=1.52$$

a. Given the refractive index of water, $\boldsymbol{n_w = n_1}$

By using the lens maker's equation, thus:

$$\frac{1}{f} = (\frac{n_g}{n_w} - 1)(\frac{1}{r_1} + \frac{1}{r_2})$$

$$\boxed{f = +70\ cm}$$

b. Given the refractive index of carbon disulfide, $\boldsymbol{n_c = n_1}$

By using the lens maker's equation, thus:

$$\frac{1}{f} = (\frac{n_g}{n_c} - 1)(\frac{1}{r_1} + \frac{1}{r_2})$$

$$\boxed{f = -148.18\ cm}$$

Example:

An object is placed $(90cm)$ from a glass lens $(n = 1.56)$ with one concave surface of radius $(22cm)$ and one convex surface of radius $(18.5cm)$. Determine:

a. The image position.

b. The linear magnification.

Solution:

$$u=+90.0\ cm,\ n=1.56,\ r_1=-22.0\ cm,\ r_2=+18.5\ cm$$

a. By applying the lens maker's equation in air:

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$\boxed{f=+208\ cm}$$

By applying the thin lens formula, thus:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad ...(5)$$

$$\boxed{v=-159\ cm}$$

The image forms $(159cm)$ in front of the lens (at the same side of the object placed).

b. By applying equation of linear magnification for thin lens, thus:

$$M=-\frac{v}{u}$$

$$\boxed{\therefore M=1.77}$$