Kirkuk University

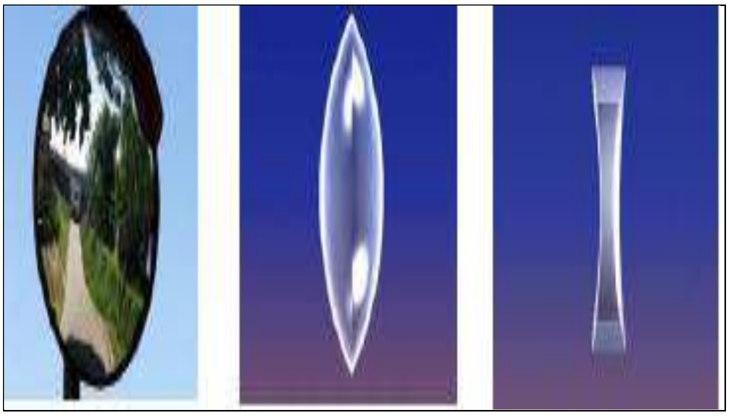
Science College

Physics Department

Lectures of

GEOMETRIC OPTICS

Lecture – 17 –



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Lecture 17: Refraction of Spherical Surfaces

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17-1 Refraction of Spherical Surfaces

- As we mentioned in past lectures, images can be formed by reflection as well by refraction.
- In this lecture we consider refraction at a spherical surface that is at a spherical interface between two optical materials with different indexes of refraction.
- Fig.(**17-1**) below shows a spherical surface with radius, ***r*** forms an interface between two media with refractive indices n_1 and n_2.

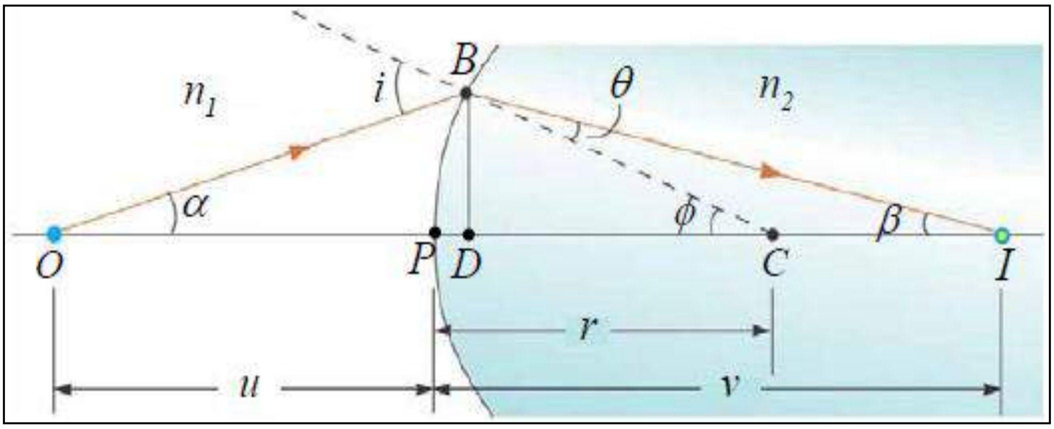


Fig.(17-1) : Refraction of Spherical Surface

- The surface forms an image ***I*** of a point object ***O*** as shown in figure above.
- The incident ray ***OB*** making an angle ***i*** with the normal and is refracted to ray ***BI*** making an angle **θ** where $n_1<n_2$.
- Point ***C*** is the centre of curvature of the spherical surface and ***BC*** is normal.

- By using the theorem that an exterior angle of a triangle equals the sum of the two opposite interior angles , applying this to the triangles **Δ *BOC*** , *and* **Δ *BIC*** from the figure gives **:**

$$\Delta\, BOC \Rightarrow \boxed{i = \alpha + \phi ...(1)}$$

$$\Delta\, BIC \Rightarrow \phi = \beta + \theta \Rightarrow \boxed{\theta = \phi - \beta ...(2)}$$

- From the **Snell's law :**

$$n_1.\sin\theta_1 = n_2.\sin\theta_2$$

- By using **Δ BOD**, **Δ BCD** and **Δ BID** thus **:**

$$\tan\alpha = \frac{BD}{OD}, \tan\phi = \frac{BD}{CD}, \tan\beta = \frac{BD}{ID}$$

- By considering point **B** very close to the pole **P**, hence **:**

$$\sin i \approx i, \sin\theta \approx \theta, \tan\alpha \approx \alpha, \tan\phi \approx \phi, \tan\beta \approx \beta$$
$$OD \approx OP = d_o, CD \approx CP = r, ID \approx IP = d_i$$

- Then **Snell's law** can be written as **:**

$$\boxed{n_1.i = n_2.\theta ...(3)}$$

- By substituting eq. (**1**) and (**2**) into eq. (**3**), thus **:**

$$n_1(\alpha + \phi) = n_2(\phi - \beta)$$
$$n_1.\alpha + n_2.\beta = (n_2 - n_1)\phi$$

- Then **:**

$$n_1(\frac{BD}{d_o}) + n_2(\frac{BD}{d_i}) = (n_2 - n_1)(\frac{BD}{r})$$

$$\boxed{\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{(n_2 - n_1)}{r} \ldots(4)}$$

- Then equation (**4**) represented equation of spherical refracting surface , where **:**

  d_o : Object distance from pole**.**

  d_i : Image distance from pole**.**

  n_1 : Refractive index of medium **1** (Medium containing the incident ray)**.**

  n_2 : Refractive index of medium **2** (Medium containing the refracted ray)**.**

  r : Radius of spherical surface**.**

- Note , If the refraction surface is flat (**plane**) , $r = \infty$ then **:**

$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = 0$$

- The equation (formula) of linear magnification for refraction by the spherical surface is given by **:**

$$\boxed{M = \frac{h_i}{h_o} = -\frac{n_1 d_i}{n_2 d_o} \ldots(5)}$$

17 – 2 The +/- Sign Conventions for Refraction Of Spherical Surface

- The sign conventions for the given quantities in the equation of spherical refracting surface and magnification equations are as follows:

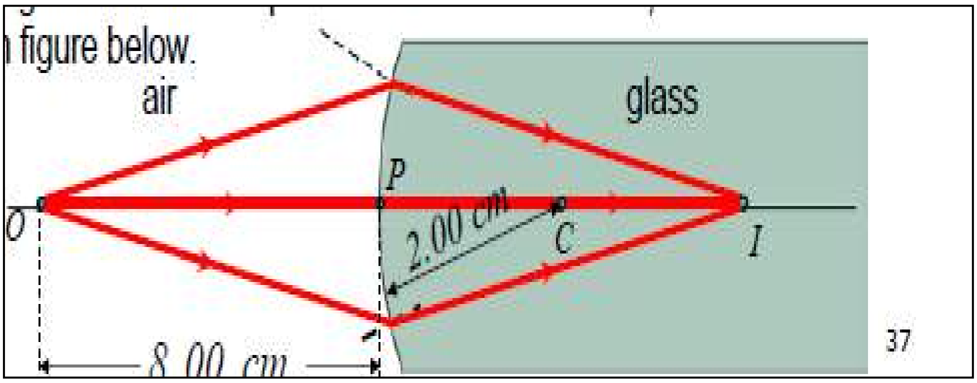
- (f) is $(+)$ if the spherical surface is a (convex surface).
- (f) is $(-)$ if the spherical surface is a (concave surface).
- (d_i) is $(+)$ if the image is a real image and located on the opposite side of the object.
- (d_i) is $(-)$ if the image is a virtual image and located on the same side of the object.
- (M) is $(+)$, image is upright (erect).
- (M) is $(-)$, image is inverted.
- (r) is $(+)$, if the spherical surface is a (convex surface).
- (r) is $(-)$, if the spherical surface is a (concave surface).

Example:

A cylindrical glass rod in air has refractive index of (1.52). One end is ground to a hemispherical surface with radius, $(r = 2cm)$ as shown in figure below. Find:

a. The position of the image for a small object on the axis of the rod, $(8cm)$ to the left of the pole as shown in figure.

b. The linear magnification. (Given the refractive index of air, $(n_a = 1)$).



Solution: $(n_a = n_1 = 1)$, $(n_g = n_2 = 1.52)$, $(d_o = 8cm)$, $(r = +2cm)$

a. By applying the equation of spherical refracting surface:

$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{(n_2 - n_1)}{r} \quad ...(4)$$

$$\frac{n_a}{d_o} + \frac{n_g}{d_i} = \frac{(n_g - n_a)}{r}$$

$$\boxed{d_i = +11.26cm}$$

The image is $(11.26cm)$ at the back of the convex surface.

b. By using the equation of linear magnification for refracting surface**:**

$$M = \frac{h_i}{h_o} = -\frac{n_1 d_i}{n_2 d_o} \quad ...(5)$$

$$M = -\frac{n_a d_i}{n_g d_o}$$

$$\boxed{\Rightarrow M = -0.93}$$

Negative sign indicates the image is **inverted.**

Example:

A point object is $(25cm)$ from the center of a glass sphere of radius $(5cm)$. The refractive index of glass is (1.50). Find the position of the image formed due to refraction by**:**

a. The first spherical glass surface**.**

b. The first and second refractive surfaces of spheres**.**

Solution:

a. Given $(n_a = n_1 = 1)$, $(n_g = n_2 = 1.50)$, $(d_o = 20cm)$, $(r = 5cm)$.

By using the equation of spherical refracting surface, thus**:**

$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{(n_2 - n_1)}{r} \quad ...(4)$$

For first surface (**convex surface**):

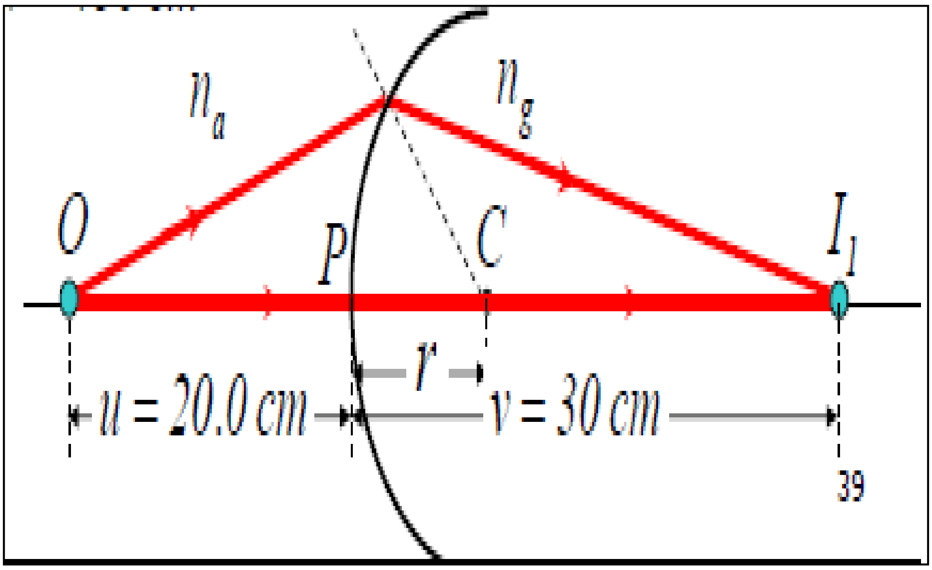
$$r = +5.0cm$$

$$\frac{n_a}{d_o} + \frac{n_g}{d_i} = \frac{(n_g - n_a)}{r}$$

$$\frac{1}{20} + \frac{1.5}{d_i} = \frac{(1.5-1)}{5}$$

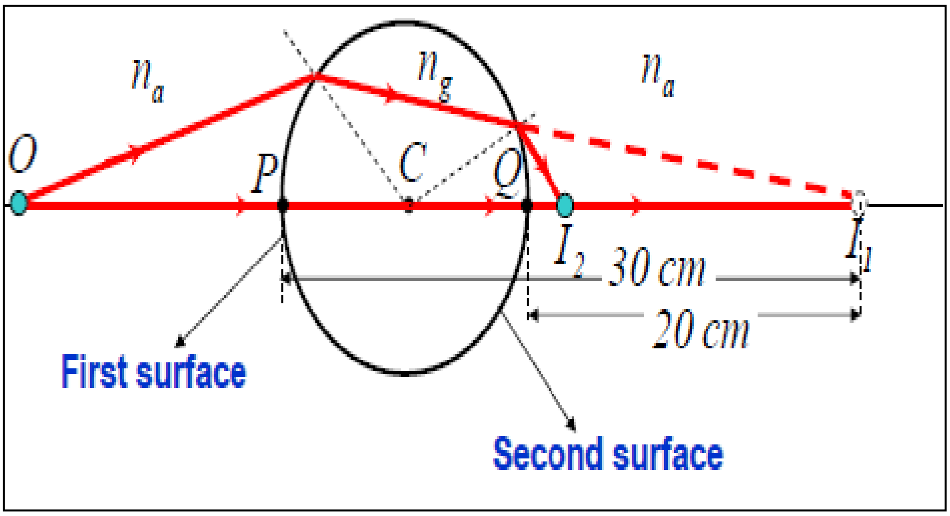
$$\boxed{d_i = +30cm}$$

The image is real and $(30cm)$ at the back of the convex surface.



b. From the figure below, the image I_1 formed by the first surface is in glass and $(20cm)$ from the point **Q** of the second surface**.**

I_1 acts as a **virtual object** for the second refraction surface and**:**



$$(n_g = n_1 = 1.5)\,,(n_a = n_2 = 1)\,,(d_o = -20cm)\,,(r = 5cm)$$

For second surface (**concave surface**)**:**

$$r = -5.0cm$$

$$\frac{n_g}{d_o} + \frac{n_a}{d_i} = \frac{(n_a - n_g)}{r}$$

$$\frac{1.5}{-20} + \frac{1}{d_i} = \frac{(1-1.5)}{-5}$$

$$\boxed{d_i = +5.71cm}$$

The image is real and $(5.71cm)$ at the back of the concave surface ($(5.71cm)$ from point **Q** as shown in figure above)**.**