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Lecture 8 – WIND – PART 2

8.1 GEOSTROPHIC BALANCE

- Above the planetary boundary layer we can ignore friction
- This leaves only two forces, **PGF** and **COR**
- These two forces are usually very close to being in balance
- This means that they must be acting in opposite directions
- Since **COR** is always to the right of the direction of motion, then the **PGF** must be to the left as shown in figure (**8.1**)
- This means that:
  - THE WIND BLOWS ALONG LINES OF CONSTANT PRESSURE, WITH LOWER PRESSURE TO THE LEFT

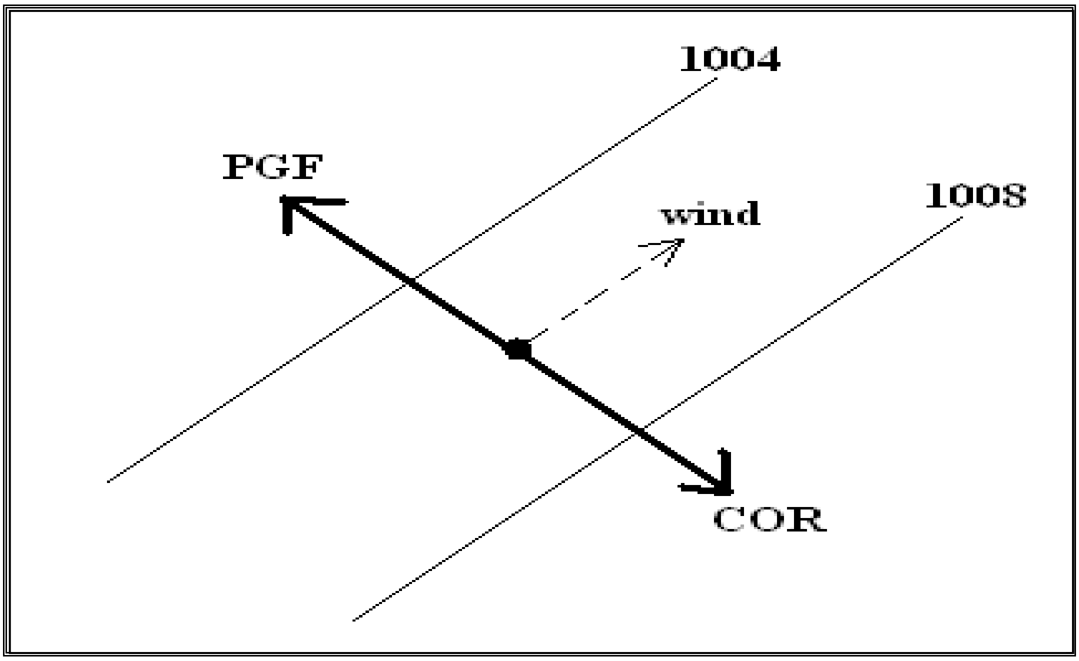

Fig.(8.1) : Force balance for geostrophic flow.

Note that PGF and COR are equal and opposite.

- The balance between pressure gradient force and Coriolis force is called *geostrophic balance*
- If the pressure gradient is tighter, the **PGF** will be stronger
- This means the Coriolis force will also have to be stronger to balance the **PGF** as shown in figure (**8.2**)
- The only way to increase the **COR** is to speed up the wind
- Thus:
  - A TIGHTER PRESSURE GRADIENT MEANS FASTER WINDS

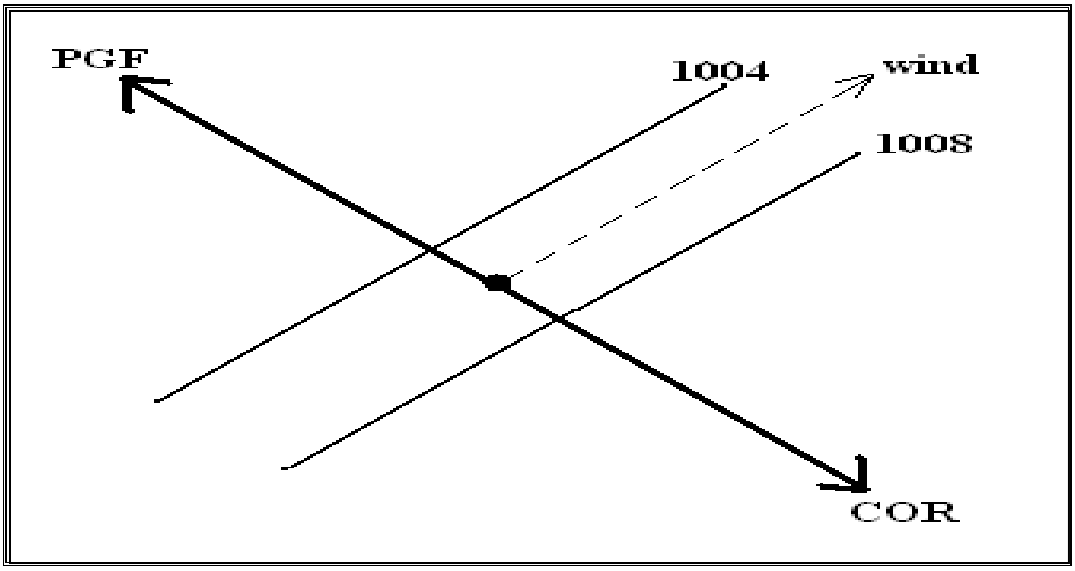

Fig. (8.2) : A tighter pressure gradient means larger PGF which requires larger COR to balance it. Larger COR implies faster wind speed.

- ***BUYS BALLOTT'S LAW*** – In the northern hemisphere, if you stand with your back to the wind, low pressure will be to your left
- Properties of the geostrophic wind are:
  - The geostrophic wind blows parallel to the isobars with lower pressure to the left
  - The greater the pressure gradient, the stronger the geostrophic wind
  - A given pressure gradient will give a stronger geostrophic wind at lower latitudes (because *f* gets smaller at lower latitudes)
- The magnitude (or speed) of the geostrophic wind is

$$V_g = \frac{1}{\rho f}\left|\nabla p\right| \qquad V_g \cong \frac{1}{\rho f}\left|\frac{\Delta p}{\Delta n}\right|$$

8.2 CURVED FLOW (THE GRADIENT WIND)

- In geostrophic flow the pressure gradient force exactly balances the Coriolis force
- However, if the atmosphere were exactly in geostrophic balance, then the flow would always be in a straight line
- This is because curved flow is accelerating (changing directions)
- We know that flow in the atmosphere is often curved
- There is counterclockwise flow around low-pressure systems, and clockwise flow around high pressure systems as shown in figure (**8.3**)

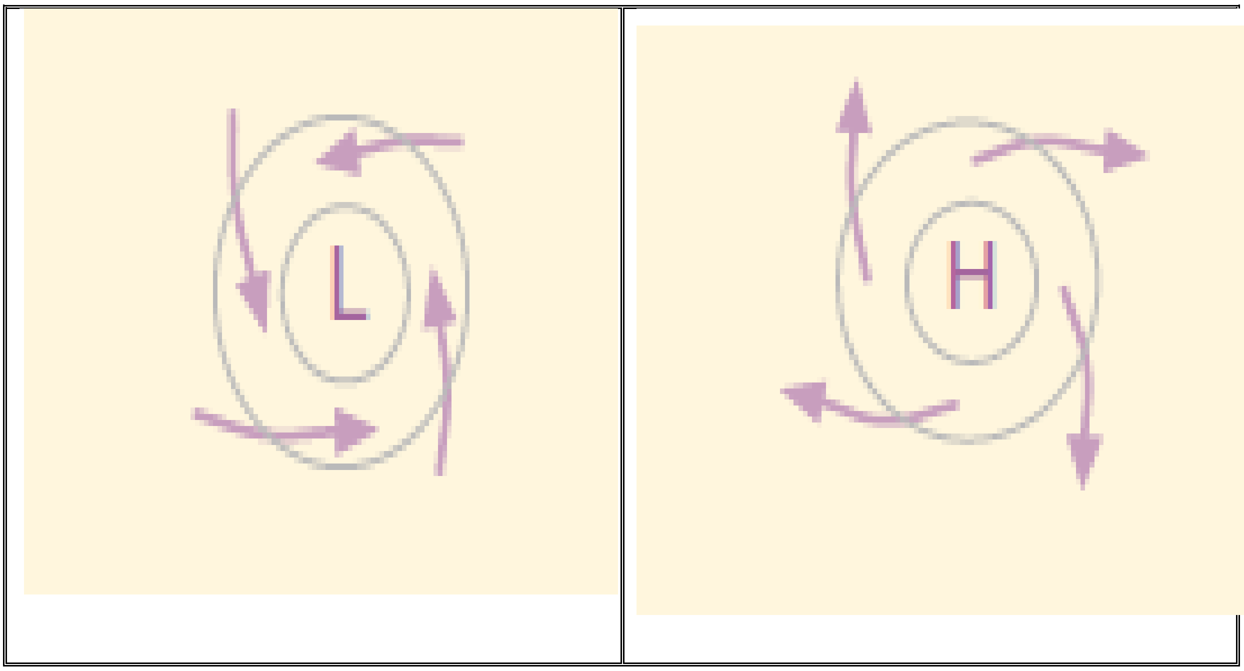

Fig. (8.3) : Winds motions around low-pressure systems and high – pressure system in the Northern Hemisphere.

- Counterclockwise flow is known as *cyclonic flow*, so low-pressure systems are known as cyclones
- Clockwise flow is known as *anticyclonic flow*, so high-pressure systems are known as anticyclones
- In curved flow, there must be a net force toward the center of the curvature
- The pressure gradient force and Coriolis force can't balance in curved flow
- The larger of the two forces must point toward the center of the curvature

- In cyclonic flow the pressure gradient force is stronger than the Coriolis force as shown in figure (**8.4**)

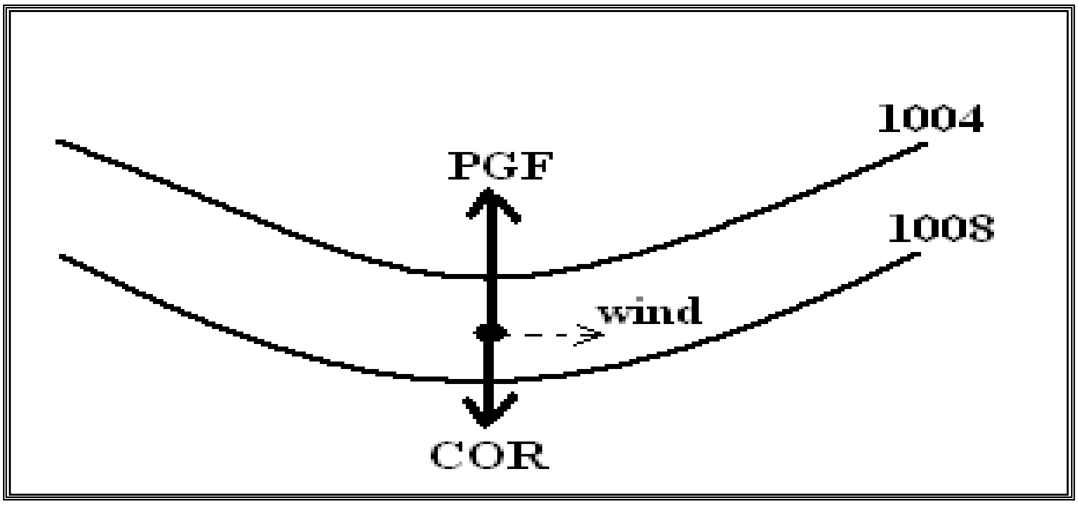

Fig.(8.4) : Force balance in cyclonic flow. Larger force must point toward center of curve. Therefore, in cyclonic flow the PGF is larger than the COR

- In anticyclonic flow the pressure gradient force is weaker than the Coriolis force as shown in figure (**8.5**)

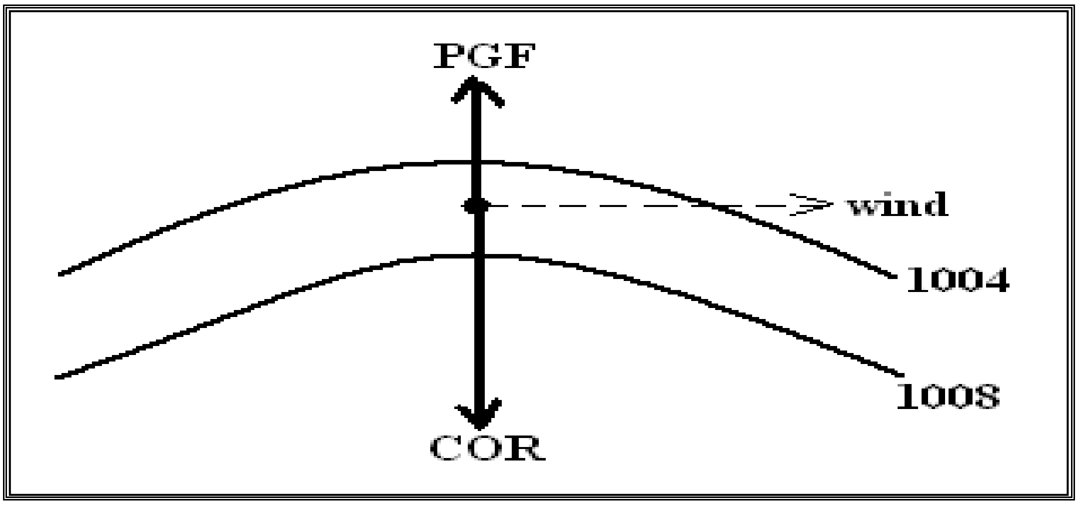

Fig.(8.5) : Force balance in anticyclonic flow. Larger force must point toward center of curve. Therefore, in anticyclonic flow the COR is larger than the PGF

- The flow around curved isobars is called *gradient flow*
- Assuming that the pressure gradient remains the same, the flow around an anticyclone will be stronger than straight flow, and the flow around a cyclone will be weaker than for straight flow as shown in figure (**8.6**)

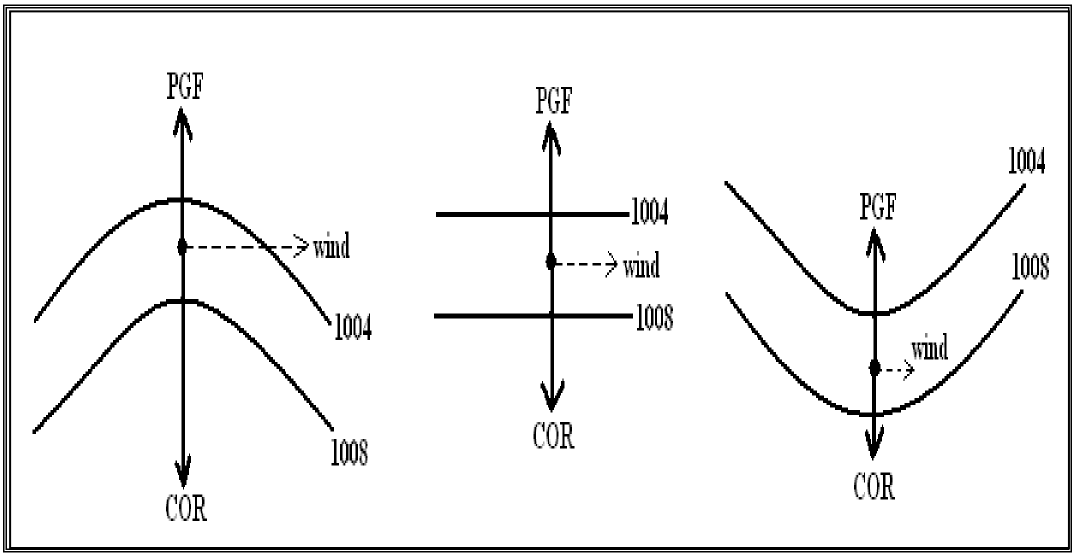

Fig . (8.6) : Comparison of force balance and wind for anticyclonic, straight (geostrophic), and cyclonic flow.

Note that the PGF is the same in all three cases, since the isobar spacing is the same. But, the COR (and hence, the wind speed) is different in each case

8.3 THE HYPSOMETRIC EQUATION

- The atmosphere can often be considered to be in hydrostatic balance

$$\frac{dp}{dz} = -\rho g$$

- In terms of geopotential height the hydrostatic equation is

$$\frac{dp}{dz} = -\rho g_{\circ}$$

- Substituting from the ideal gas law

$$p = \rho R_d T \qquad \Rightarrow \rho = \frac{p}{R_d T}$$

- We have

$$\frac{dp}{dZ} = -\frac{p g_{\circ}}{R_d T}$$

OR

$$dZ = -\frac{R_d}{g_{\circ}} \frac{T}{P} dp$$

- Integrating between two levels in the atmosphere gives

$$\int_{Z_1}^{Z_2} dZ = -\frac{R_d}{g_{\circ}} \int_{z_1}^{Z_2} \frac{T}{p} \frac{dp}{dZ} dZ$$

OR

$$Z_2 - Z_1 = -\frac{R_d}{g_{\circ}} \int_{P_1}^{P_2} T \frac{dp}{p}$$

- Using the generalized mean value theorem of calculus this becomes

$$Z_2 - Z_1 = -\frac{R_d}{g_\circ}\overline{T}\int_{P_1}^{P_2}\frac{dp}{p}$$

- Where T' is the average temperature in the layer between $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$
- The formula for the geopotential distance between the two pressure levels is

$$Z_\Delta = Z_2 - Z_1 = \frac{R_d}{g_\circ}\overline{T}\ln(p_1 / p_2)$$

and is called the *hypsometric equation*

- The hypsometric equation tells us that the *thickness* or difference in geopotential height ($\boldsymbol{Z_\Delta}$) between two pressure levels is proportional to the average temperature of the layer between the two levels
  - A colder average temperature equals a smaller thickness
  - A warmer average temperature equals a larger thickness

8.4 THE THERMAL WIND

- The *thermal wind* is defined as the vector difference in the geostrophic wind between two levels of the atmosphere

$$\vec{V}_T = \vec{V}_{g2} - \vec{V}_{g1}$$

- The thermal wind tells us how the geostrophic wind changes with height
- The geostrophic wind in pressure coordinates is

$$\vec{V}_g = \frac{g_\circ}{f}\hat{k}x\nabla Z$$

Therefore, the thermal wind is

$$\vec{V}_T = \frac{g_\circ}{f}\hat{k}x\nabla Z_2 - \frac{g_\circ}{f}\hat{k}x\nabla Z_1$$

$$\vec{V}_T = \frac{g_\circ}{f}\hat{k}x\nabla(Z_2 - Z_1)$$

$\boldsymbol{Z_2 - Z_1}$ is just the thickness, Z_Δ, of the layer.

Therefore, the thermal wind is

$$\vec{V}_T = \frac{g_\circ}{f}\hat{k}xZ_\Delta$$

- This equations tells us that *the thermal wind will be oriented parallel to the thickness contours, with low thickness to the left*

- Using the hypsometric equation ,

$$Z_{\Delta} = Z_2 - Z_1 = \frac{R_d}{g_{\circ}} \overline{T} \ln(p_1 / p_2)$$

- We can write the thermal wind equation in terms of the layer-average temperature gradient as

$$\vec{V}_T = \frac{R_d}{f} \ln(p_1 / p_2) \hat{k} x \nabla \overline{T}$$

- Since thickness is a measure of the average temperature of the layer, *the thermal wind will be oriented with lower temperatures to the left*
- The rules for the thermal wind are:
    - Blows parallel to the thickness lines with low temperatures to the left (Northern Hemisphere)
    - Tighter thickness gradient leads to stronger thermal wind